Measuring eccentricity from gravitational waveform

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Outline

Need for Eccentric Waveforms

Current state of eccentric waveform models

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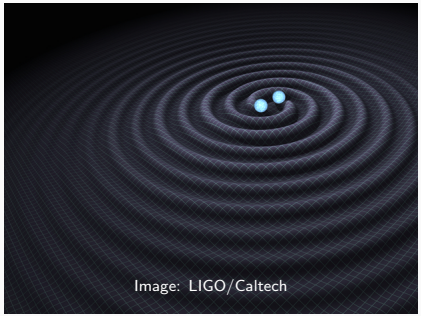
Summary and Remarks

Need for Eccentric Waveforms

So far GW data analysis use quasicircular waveforms

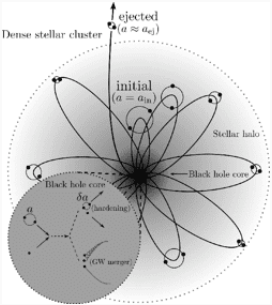


- About 90 CBCs have been detected.
- Analysed using quasicircular waveform models $\rightarrow$ eccentricity $= 0$
- Assuming that binaries circularize before entering detector band
- Binary lose eccentricity as it inspirals

[Peters and Mathews, 1963, Peters, 1964]

The case for eccentricity

- Binaries formed in galactic fields → isolated evolution → circularization
- Dynamical formation → highly eccentric binary [Mapelli, 2020]
  - globular cluster via direct capture [Rodriguez et al., 2019, Rodriguez et al., 2018, Rodriguez et al., 2016, Samsing et al., 2014, Samsing et al., 2018]
  - galactic center [Antonini and Rasio, 2016]
  - Field triples via Kozai-Lidov oscillation [Naoz, 2016, Antonini et al., 2017]
- Require eccentric model for detection and analysis of these signals.



J. Samsing (2017)

Current state of eccentric waveform models

Existing models

- Post-Newtonian
  - `EccentricTD` [Tanay et al., 2016]
  - `EccentricFD` [Huerta et al., 2014]
- Effective One Body
  - `SEOBNRE` [Cao and Han, 2017, Liu et al., 2020] `SEOBNREHM` [Liu et al., 2022]
  - `SEOBNRv4EHM` [Ramos-Buades et al., 2022a]
  - `TEOBResumS` [Nagar et al., 2021, Chiaramello and Nagar, 2020, Nagar et al., 2018]
- Numerical Relativity
  - `SpEC` [SXS Collaboration,]
  - `RIT` [Healy and Lousto, 2022]
- Numerical Relativity Surrogate $\rightarrow$ `NRSur2dq1Ecc` [Islam et al., 2021]
  `NRSur3dq4Ecc` (ongoing)

PE with eccentric models

- Non-LIGO works claiming evidence eccentricity [Gayathri et al., 2022, Romero-Shaw et al., 2022b, Romero-Shaw et al., 2020, Romero-Shaw et al., 2019]
- Degeneracy between eccentricity and precession or correlation with other parameters [Romero-Shaw et al., 2022a, O'Shea and Kumar, 2021]
- Test of GR Bias due to neglecting eccentricity.

A few issues with current models

- Incompatible definitions of eccentricity → model dependence [Knee et al., 2022]
- No unique definition of eccentricity in GR → gauge dependence
- Neglecting mean anomaly as a free parameter [Islam et al., 2021, Clarke et al., 2022]
- Lack of a standardized definition → ambiguity in PE inference.

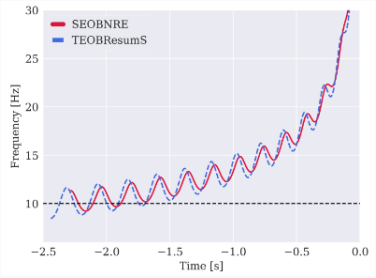


Image: A. Knee (2022)

Standardizing Eccentricity Definition

Features of eccentric orbit

- Requires two additional parameters, eccentricity e and mean anomaly l.
- Intrinsic parameters $\rightarrow$ masses, spins, eccentricity, mean anomaly.
- Maximum (minimum) radiation of GW at pericenter (apocenter) $\rightarrow$ modulations in amplitude and frequency.
- In GR, precession of pericenter $\rightarrow$ binary orbits are no longer closed ellipse.

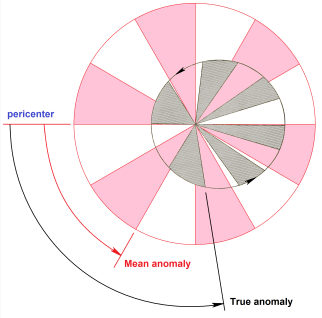


Image: Wikipedia

Defining Eccentricity: required features

- Three parameters: eccentricity e and mean anomaly l at given reference frequency $f_{\rm ref}$.
- Gauge independent and model independent
- Reduces to Keplerian eccentricity in Newtonian limit
- Applicable to full range of eccentricity $(0 - 1)$ for bound orbits.
- Applicable to waveforms of different origins.
- Computationally cheap.

Attempt 1: Using orbital frequency

Defining eccentricity using orbital frequency [Mora and Will, 2002, Lewis et al., 2017]

$$e_{\Omega_{\rm orb}}(t) = \frac{\sqrt{\Omega^{\rm p}_{\rm orb}(t)} - \sqrt{\Omega^{\rm a}_{\rm orb}(t)}}{\sqrt{\Omega^{\rm p}_{\rm orb}(t)} + \sqrt{\Omega^{\rm a}_{\rm orb}(t)}} \to \frac{r^{\rm a} - r^{\rm p}}{r^{\rm a} + r^{\rm p}} \qquad \text{reduces to correct Newtonian limit.} \tag{1}$$

$\Omega^{\rm p}_{\rm orb}/\Omega^{\rm a}_{\rm orb}$ and $r^{\rm p}/r^{\rm a} \to$ orbital frequency and separation at peri-/apocenter.

$e_{\Omega_{\rm orb}}$ is coordinate dependent since $\Omega^{\rm p}_{\rm orb}/\Omega^{\rm a}_{\rm orb}$ are obtained from the trajectories.

Attempt 2: Using waveform frequency

To remove coordinate dependence, substitute $\Omega_{\rm orb}$ with ω_{22}

[Ramos-Buades et al., 2020, Islam et al., 2021, Ramos-Buades et al., 2022a, Bonino et al., 2022]

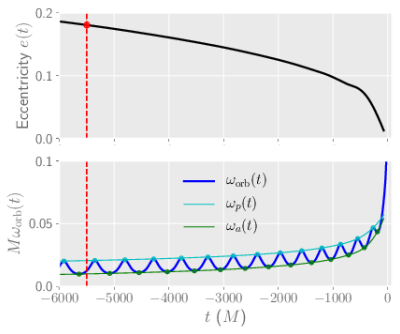


Image: Islam (2021)

$$h_+ - ih_\times = \sum_{\ell=2}^{\ell=\infty} \sum_{m=-\ell}^{m=\ell} \hbar_{\ell m}(\lambda, t)\, Y^{\ell m}_{-2} \tag{2}$$

$$e_{\omega_{22}} = \frac{\sqrt{\omega^{\rm p}_{22}(t)} - \sqrt{\omega^{\rm a}_{22}(t)}}{\sqrt{\omega^{\rm p}_{22}(t)} + \sqrt{\omega^{\rm a}_{22}(t)}} \tag{3}$$

$$\omega_{22} = \frac{{\rm d}\phi_{22}}{{\rm d}t} \tag{4}$$

$$\hbar_{22} = A_{22} e^{-\phi_{22}} \tag{5}$$

However, it does not give correct Newtonian limit $e_{\omega_{22}} = (3/4)e_t$ [Ramos-Buades et al., 2022b]

Proposed definition of eccentricity

- To get the correct limit, we define $e_{\rm gw}$ [Ramos-Buades et al., 2022b]

$$e_{\rm gw} = \cos(\Psi/3) - \sqrt{3}\sin(\Psi/3), \qquad \Psi = \arctan\left(\frac{1 - e^2_{\omega 22}}{2\, e_{\omega 22}}\right). \tag{6}$$

  $e_{\rm gw}$ reduces to correct Newtonian definition in the Newtonian limit and it is coordinate-independent.

- Mean anomaly $l_{\rm gw}$ can be defined as

$$l_{\rm gw}(t) = 2\pi \frac{t - t_p}{t_{p+1} - t_p}, \quad t_p \leq t < t_{p+1}, \tag{7}$$

  where $t_p/t_{p+1} \rightarrow$ time of previous/next pericenter passage.

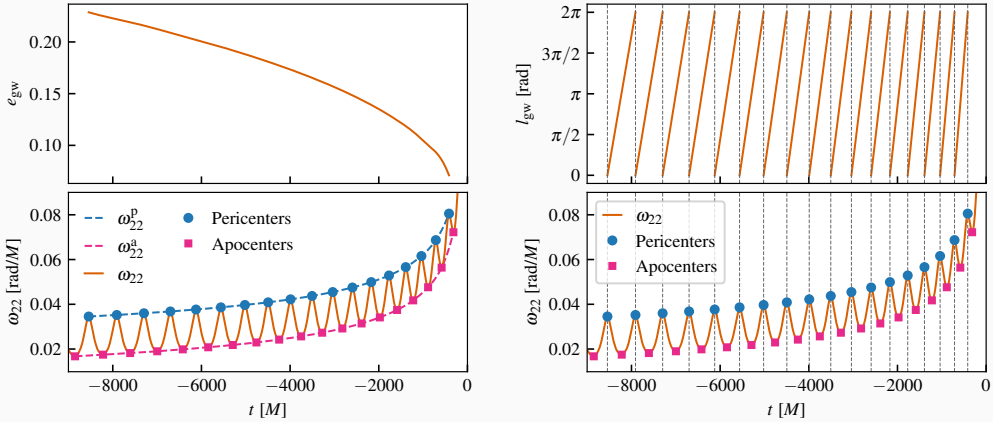


Figure 1: Standardizing eccentricity and mean anomaly definition. Shaikh+ (2023)

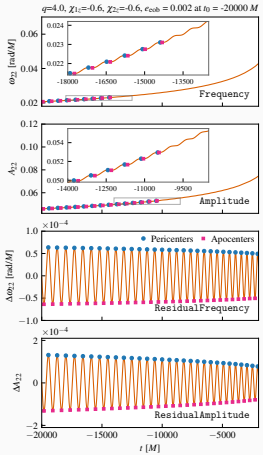
Implementation and demo

Implementation package: gw_eccentricity

- Public Python package gw_eccentricity to measure eccentricity and mean anomaly from GW waveform.
- Available in PyPi
- 6 different methods to compute eccentricity.
- Will undergo LIGO review for possible use in case of eccentric GW detection.

Eccentricity measurement methods: Amplitude and Frequency

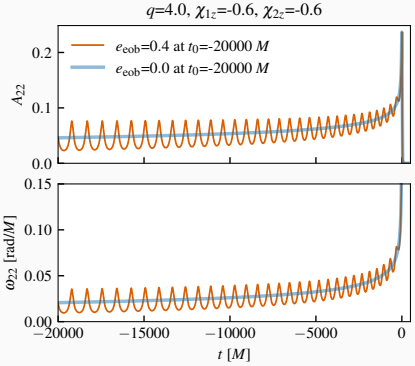
Each method is named after the data $U(t)$ it uses for finding the pericenter/apocenter.



- `Amplitude` or `Frequency` uses $U(t) = A_{22}$ or ω_{22}
- Works for only relatively large eccentricity $\gtrsim 10^{-3}$

$$e_{\rm gw} \gtrsim \frac{192}{15}\nu\left(\frac{M\omega_{22}}{2}\right)^{5/3}. \qquad (8)$$

ResidualAmplitude and ResidualFrequency



- Uses residual data
- For `ResidualFrequency`

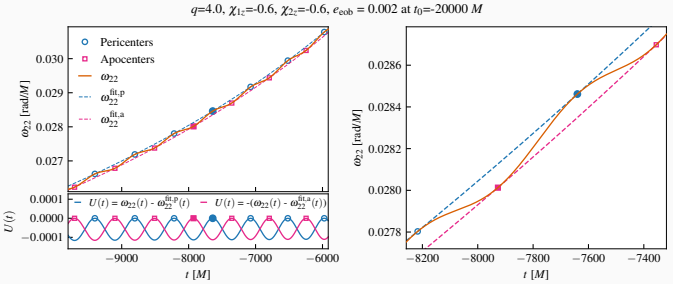
$$U(t) = \Delta\omega_{22}(t) \equiv \omega_{22}(t) - \omega_{22}^{\mathrm{circ}}(t), \qquad (9)$$

and likewise for the `ResidualAmplitude`

$$U(t) = \Delta A_{22}(t) \equiv A_{22}(t) - A_{22}^{\mathrm{circ}}(t), \qquad (10)$$

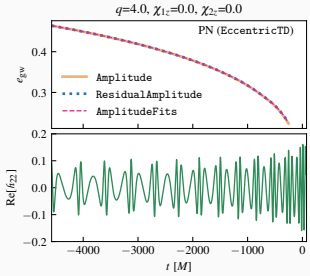
- Works for full range of $e_{\mathrm{gw}} \in (0 - 1)$

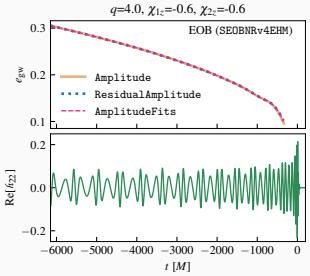
AmplitudeFits and FrequencyFits

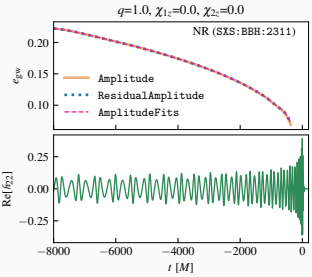


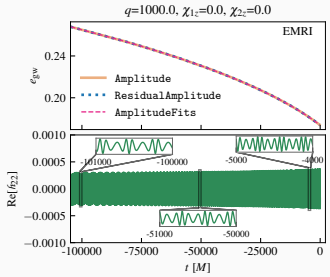
- It uses residual data $U(t) = \omega_{22}(t) - \omega_{22}^{\text{fit,p}}(t)$, where $\omega_{22}^{\text{fit,p}}(t;\, A,\, n,\, t_{\text{merg}}) = A(t_{\text{merg}} - t)^n$
- Works for full range $(0 - 1)$
- Less reliable than `ResidualAmplitude` or `ResidualFrequency`.

Applicable to different waveform models

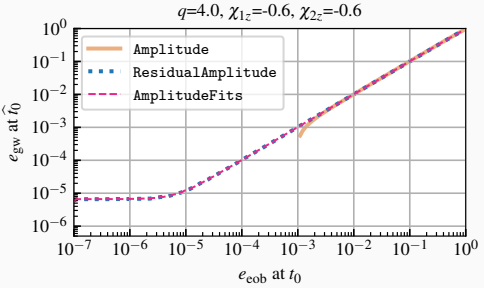








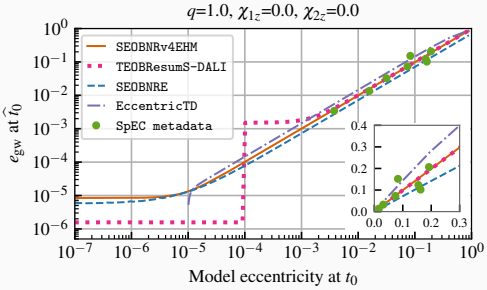
Applicable to full range of eccentricity



Shaikh+ (2023)

- Residual/Fits Can measure eccentricity $e_{\rm gw} \approx 10^{-5}$ to $e_{\rm gw} \approx 1.0$
- Amp/Freq fails for $e_{\rm gw} \lesssim 10^{-3}$
- Highlights that waveform model no longer producing distinguishable waveforms below $e_{\rm eob} \lesssim 10^{-5}$.

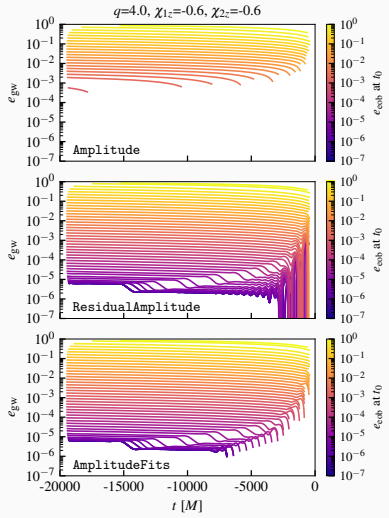
Measured eccentricity $e_{\rm gw}$ vs model eccentricity



- The models differs significantly at low eccentricity.
- `TEOBResumS-DALI` has a minimum eccentricity $10^{-4} \rightarrow e_{\rm gw} > 10^{-3}$
- `EccentricTD` has a minimum eccentricity 10^{-5}
- `SEOBNRv4EHM` and `SEOBNRE` has $e_{\rm gw} \gtrsim 10^{-5}$

Shaikh+ (2023)

Evolution of measured eccentricity $e_{\rm gw}$



- Using a set of $\approx 20,000M$ long `SEOBNRv4EHM` waveforms.
- $e_{\rm gw}$ varies smoothly with time.
- The colors represent the initial $e_{\rm eob}$ at $t_0 = -20,000M$
- `Amplitude` works for only $e_{\rm gw} \gtrsim 10^3$
- For smaller $e_{\rm eob}$, `Amplitude` stops far from the merger.
- The jumps in `ResidualAmplitude` and `AmplitudeFits` highlights issues in the waveform model.

Application in PE

Apply `gw_eccentricity` to measure eccentricity directly from waveforms at the sample parameters and reconstruct the posterior on eccentricity.

Summary and Remarks

Summary

- We implement a standardized definition of eccentricity and mean anomaly.
- This definition is model-independent, gauge-independent.
- Reduces to the well known Keplerian definition of eccentricity in the Newtonian limit.
- We provide public package `gw_eccentricity` with several methods to measure eccentricity.
- Our implementation is robust and applies to different waveform models.

Thank you!

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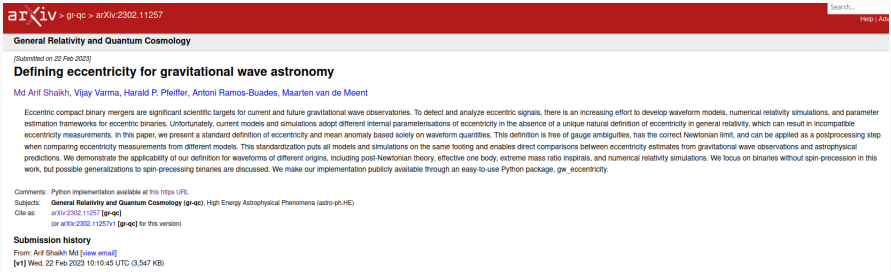

arXiv > gr-qc > arXiv:2302.11257

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General Relativity and Quantum Cosmology

[Submitted on 22 Feb 2023]

Defining eccentricity for gravitational wave astronomy

Md Arif Shaikh, Vijay Varma, Harald P. Pfeiffer, Antoni Ramos-Buades, Maarten van de Meent

Eccentric compact binary mergers are significant scientific targets for current and future gravitational wave observatories. To detect and analyze eccentric signals, there is an increasing effort to develop waveform models, numerical relativity simulations, and parameter estimation frameworks for eccentric binaries. Unfortunately, current models and simulations adopt different internal parameterisations of eccentricity in the absence of a unique natural definition of eccentricity in general relativity, which can result in incompatible eccentricity measurements. In this paper, we present a standard definition of eccentricity and mean anomaly based solely on waveform quantities. This definition is free of gauge ambiguities, has the correct Newtonian limit, and can be applied as a postprocessing step when comparing eccentricity measurements from different models. This standardization puts all models and simulations on the same footing and enables direct comparisons between eccentricity estimates from gravitational wave observations and astrophysical predictions. We demonstrate the applicability of our definition for waveforms of different origins, including post-Newtonian theory, effective one body, extreme mass ratio inspirals, and numerical relativity simulations. We focus on binaries without spin-precession in this work, but possible generalizations to spin-precessing binaries are discussed. We make our implementation publicly available through an easy-to-use Python package, gw_eccentricity.

Comments: Python implementation available at this https URL

Subjects: **General Relativity and Quantum Cosmology (gr-qc)**; High Energy Astrophysical Phenomena (astro-ph.HE)

Cite as: arXiv:2302.11257 **[gr-qc]**

(or arXiv:2302.11257v1 **[gr-qc]** for this version)

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gw-eccentricity 1.0.0

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Released: Feb 24, 2023

Defining eccentricity for gravitational wave astronomy.

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