Gravitational Wave from First Order Phase Transition during Inflation

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arXiv:2009.12381[Astro-ph.CO]

and arXiv: 2201.05171[Astro-ph.CO]

Outline

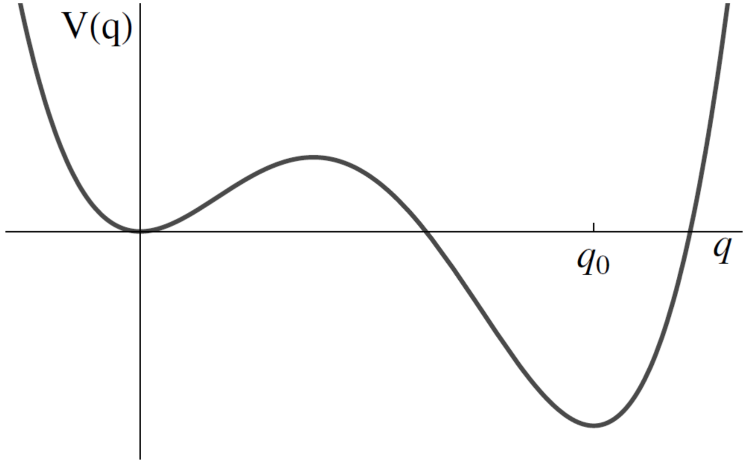
- First Order Phase Transition
- How Gravitational Wave Propagates in Different Space Time
- Observational Signatures
    - □ Stochastic Gravitational Wave Background
    - □ B mode
- Conclusion

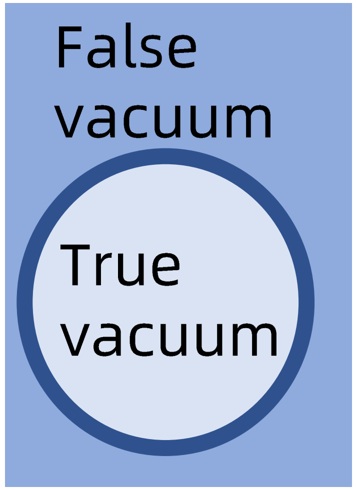
First order phase transition

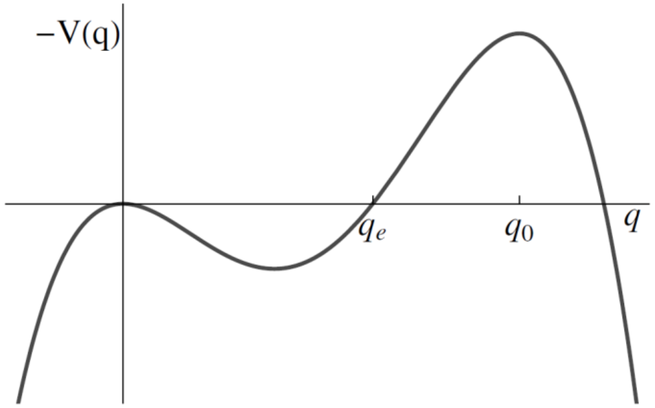
Scalar field theory in flat space

$$S_M = \int d^4x \left[-\frac{1}{2}\eta^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - V(\sigma) \right] \quad \Rightarrow \quad S_E = \int d^4x_E \left[\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + V(\sigma) \right]$$

Wick rotation

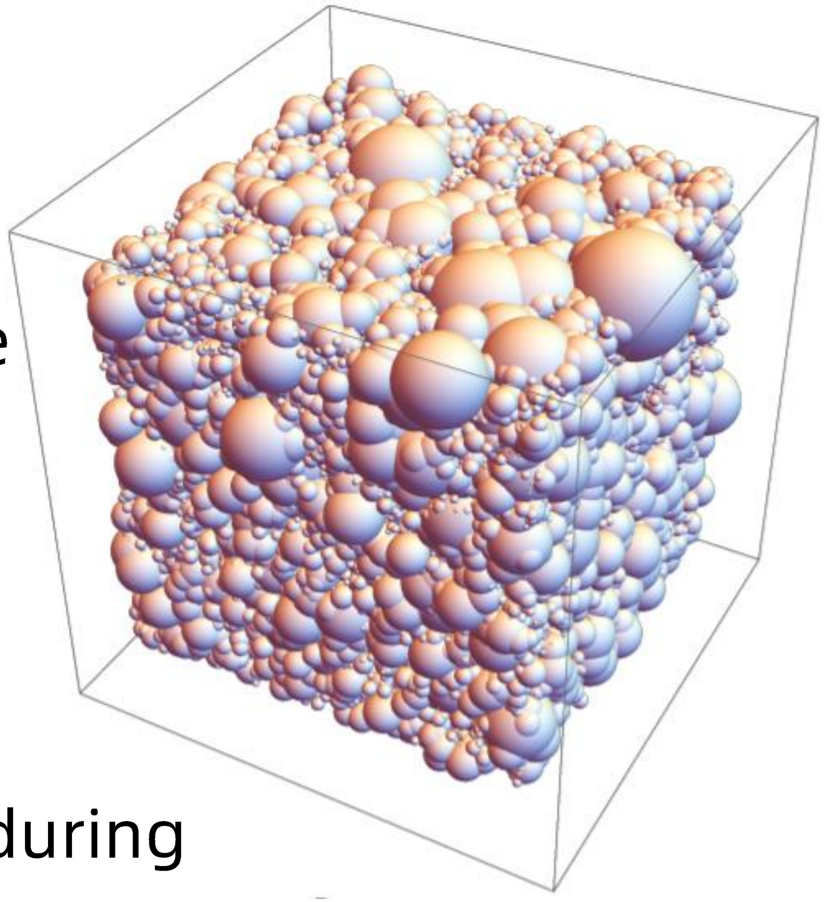






First order phase transition

- Appropriate boundary conditions
- O(4) symmetric case
- We can determine the bounce solution
- Analytical continue back to Minkowski space
- Thin-wall solution
- Bubble expands into the false vacuum
- At the speed of light
- Transition rate $\frac{\Gamma}{V} = Ae^{-S_b/\hbar}$
- Gravitational waves (GW) can be produced during the first order phase transition.



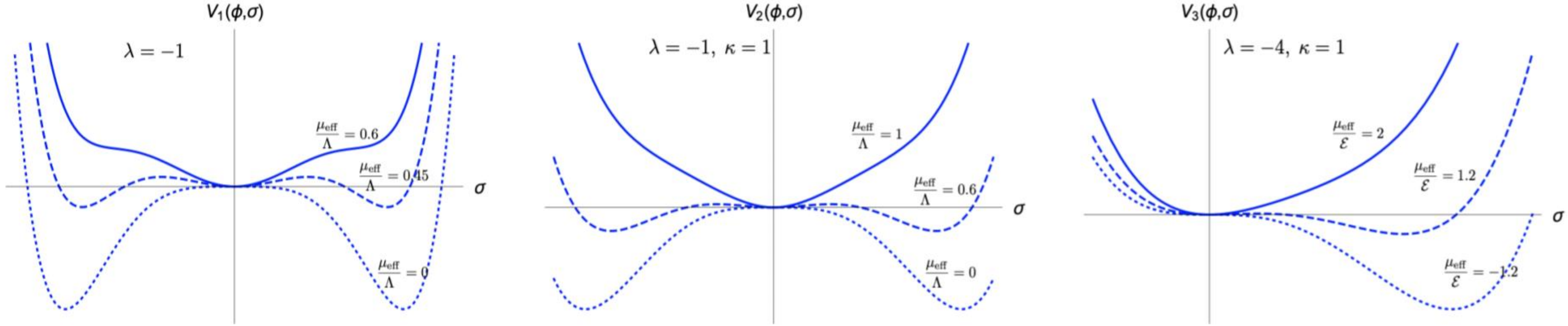
First order phase transition during inflation

Inflation sector:
dominates energy density
e.g. single field slow roll

spectator sector.
Not driving inflation

Interaction
suppressed by some high scale M

Two field case



- $\beta \equiv -dS_b/dt$ describes how fast the phase transition happens
- Determines the size of the bubble and the wavelength of the GW

$$V_1(\phi,\sigma) = -\frac{1}{2}(\mu^2 - c^2\phi^2)\sigma^2 + \frac{\lambda}{4}\sigma^4 + \frac{1}{8\Lambda^2}\sigma^6$$

$$V_2(\phi,\sigma) = -\frac{1}{2}(\mu^2 - c^2\phi^2)\sigma^2 + \frac{\lambda}{4}\sigma^4 + \frac{\kappa}{4}\sigma^4 \log\frac{\sigma^2}{\Lambda^2}$$

$$V_3(\phi,\sigma) = -\frac{1}{2}(\mu^2 - c^2\phi^2)\sigma^2 + \frac{\lambda}{3}\mathcal{E}\sigma^3 + \frac{\kappa}{4}\sigma^4$$

Typical values of β

$$\beta = \left|\frac{dS_4}{dt}\right| = \left|\frac{dS_4}{d\mu_{\rm eff}^2}\right|\left|\frac{d\mu_{\rm eff}^2}{dt}\right|$$

$$= \left|\frac{dS_4}{d\log\mu_{\rm eff}^2}\right|\left|\frac{d\mu_{\rm eff}^2}{\mu_{\rm eff}^2 dt}\right| = I_1 S_4\left|\frac{2\dot{\phi}}{\phi - \frac{\mu^2}{c^2\phi}}\right|$$

$$I_1 = \frac{1}{S_4}\left|\frac{dS_4}{d\log\mu_{\rm eff}^2}\right|$$ 0.2~5

CosmoTransitions

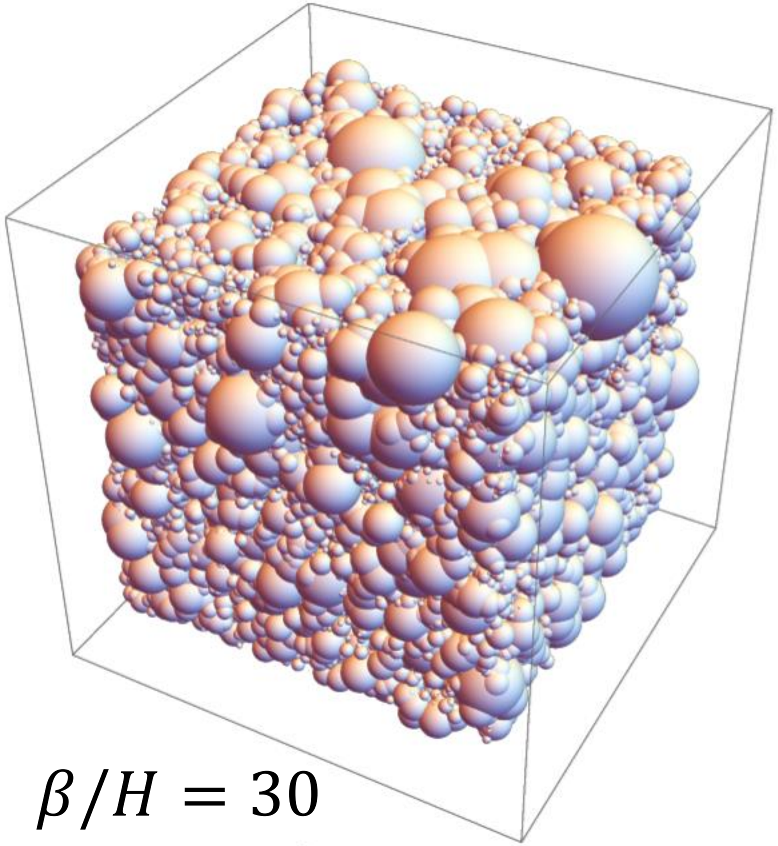
$$\frac{\beta}{H} = I_1 S_4 (2\epsilon)^{1/2} \times \frac{M_{\rm pl}}{\left|\phi - \frac{\mu^2}{c^2\phi}\right|}$$

Slow roll inflation β=O(10)~O(100)

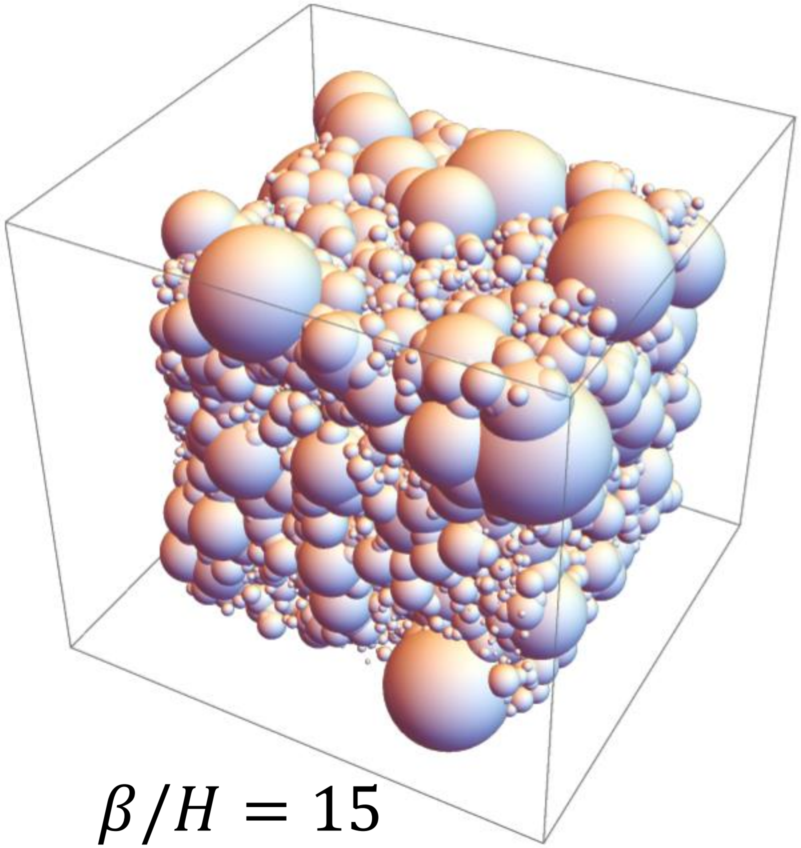
Bubble configurations

At the end of phase transition

$$r_{\text{bubble}} \equiv \beta^{-1} \sim \left(10^{-2} - 10^{-1}\right) \times H_\star^{-1}$$



$\beta/H = 30$



$\beta/H = 15$

GW from first order phase transition during inflation

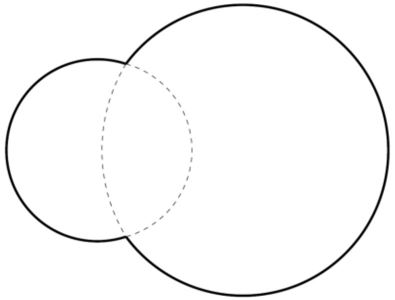
- First order phase transition happen during inflation
- The first order phase transition will produce bubbles
- Bubbles will collide and generate gravitational waves
- IR
- Intermediate
- UV

Instantaneous source

$$\frac{d\rho_{\rm GW}}{d\log k} \sim k^{-4} \frac{d\rho_{\rm GW}^{\rm flat}}{d\log k_p}$$

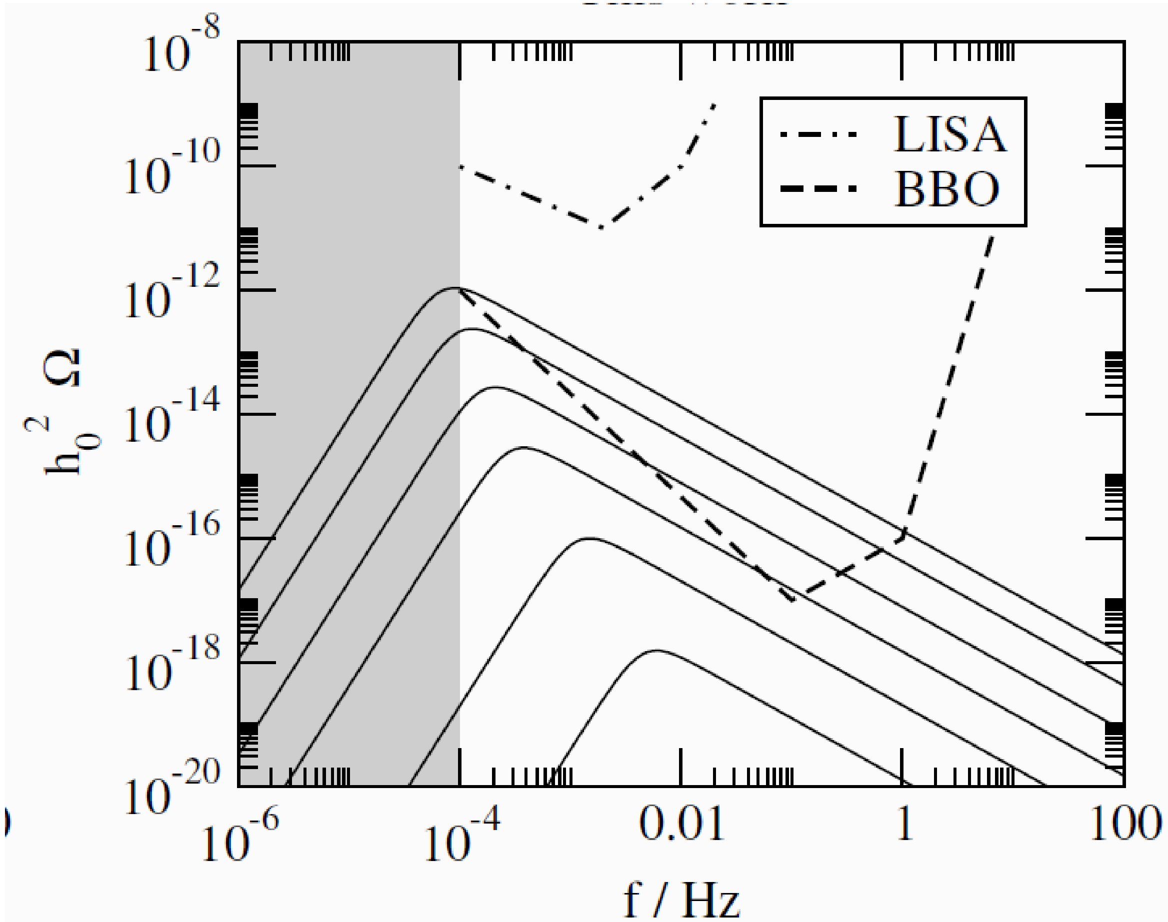
GW from first order phase transition after inflation

- Envelope approximation



$$\frac{d\rho_{\rm GW}^{\rm fat}}{\Delta\rho_{\rm vac} d\log k_p} = \kappa^2 \left(\frac{H_{\rm inf}}{\beta}\right)^2 \Delta(k_p/\beta)$$

$$\Delta(k_p/\beta) = \tilde{\Delta} \times \frac{3.8\tilde{k}_p k_p^{2.8}}{\tilde{k}_p^{3.8} + 2.8 k_p^{3.8}}$$



How GW propagates in spacetime

- Greens function $h''_{ij} + \frac{2a'}{a} h'_{ij} - \nabla^2 h_{ij} = 16\pi G_N a^2 \sigma_{ij}$ source
- Redefine variable $\mathrm{h}_{ij}(\tau, \mathrm{x}) = a(\tau) h_{ij}(\tau, \mathrm{x})$
- The new variable satisfies $\mathrm{h}''_{ij} - \left(\nabla^2 + \frac{a''}{a}\right) \mathrm{h}_{ij} = 16\pi G_N a^3 \sigma_{ij}$
- The solution is $\mathrm{h}_{ij}(\tau, \mathrm{x}) = \int d\mathrm{x}' d\tau' G_R(\tau, \tau'; \mathrm{x} - \mathrm{x}') 16\pi G_N a^3(\tau') \sigma_{ij}(\tau', \mathrm{x}')$

Green's function

- the retarded Green's function satisfies

$$\left[\frac{\partial^2}{\partial\tau^2} - \nabla^2 - \frac{a''}{a}\right] G_R(\tau,\tau';\mathbf{x}-\mathbf{x}') = \delta(\tau-\tau')\delta^3(\mathbf{x}-\mathbf{x}')$$

- Fourier transform $\tilde{G}''_R(\tau,\tau';\mathbf{k}) + \left(k^2 - \frac{a''}{a}\right)\tilde{G}_R(\tau,\tau';\mathbf{k}) = \delta(\tau-\tau')$
- Define $\tilde{\mathcal{G}}(\eta,\eta') = k\tilde{G}_R(\tau,\tau';\mathbf{k})$

$$\left(\frac{d^2}{d\eta^2} + 1 - \frac{d^2 a}{a d\eta^2}\right)\tilde{\mathcal{G}}(\eta,\eta') = 0 \qquad \tilde{\mathcal{G}}(\eta',\eta') = 0\ , \quad \left.\frac{d\tilde{\mathcal{G}}(\eta,\eta')}{d\eta}\right|_{\eta=\eta'} = 1$$

$$\left.\frac{\tilde{\mathcal{G}}(\eta,\eta')}{a(\eta)}\right|_{\eta\to 0} = \cos(\eta'-\eta_0)\tilde{\mathcal{G}}_0^f$$

During inflation

Green’s function

- Green’s function $\tilde{h}''_{ij}(\tau, \mathbf{k}) + \frac{2a'}{a}\tilde{h}'_{ij}(\tau, \mathbf{k}) + k^2\tilde{h}_{ij}(\tau, \mathbf{k}) = 0$
- Define $\tilde{h}_{ij}(\tau, \mathbf{k}) = \tilde{h}^f_{ij}(\mathbf{k})\mathcal{E}(k\tau)$
- Boundary condition $\mathcal{E}(0) = 1 \ , \quad \mathcal{E}'(0) = 0$
- Solution $\mathcal{E}(\eta) = \tilde{\mathcal{E}}^i_0 a^{-1} \sin(\eta + \phi)$

After inflation

Examples

- During inflation

  - ☐de Sitter $a(\tau) = -\frac{1}{H\tau}$

$$\tilde{\mathcal{G}}_0^f = \left(-\frac{H}{k}\right)$$

  - ☐Alternative $a(t) = \bar{a}\left(\frac{t}{\bar{t}}\right)^p = \bar{a}\left(\frac{\tau}{\bar{\tau}}\right)^{\frac{p}{1-p}}$

$$\tilde{\mathcal{G}}_0^f(k) = \left(\frac{p}{p-1}\right)^{-\frac{p}{p-1}} \bar{a}^{-1}\left(\frac{k}{\bar{a}\bar{H}}\right)^{-\frac{p}{p-1}} \frac{2^{\frac{p}{p-1}}}{\sqrt{\pi}}\Gamma\left(\frac{3}{2}+\frac{1}{p-1}\right)$$

- After inflation $a(t) \propto t^{\tilde{p}}$

$$\mathcal{E}(\eta) = \Gamma(\tilde{\alpha})^{-1}\sqrt{\pi}2^{\frac{1}{2}-\tilde{\alpha}}\csc(\pi\tilde{\alpha})\left(\eta - g\eta_{\rm end}\right)^{\tilde{\alpha}-\frac{1}{2}}\sin\left(\frac{\pi\tilde{\alpha}}{2}+\eta-g\eta_{\rm end}+\frac{\pi}{4}\right)$$

$$\tilde{\mathcal{E}}_0^i(k) = 2^{\frac{1}{2}-\tilde{\alpha}}\sqrt{\pi}f\csc(\pi\tilde{\alpha})k^{\tilde{\alpha}-\frac{1}{2}}\Gamma(\tilde{\alpha})^{-1}$$

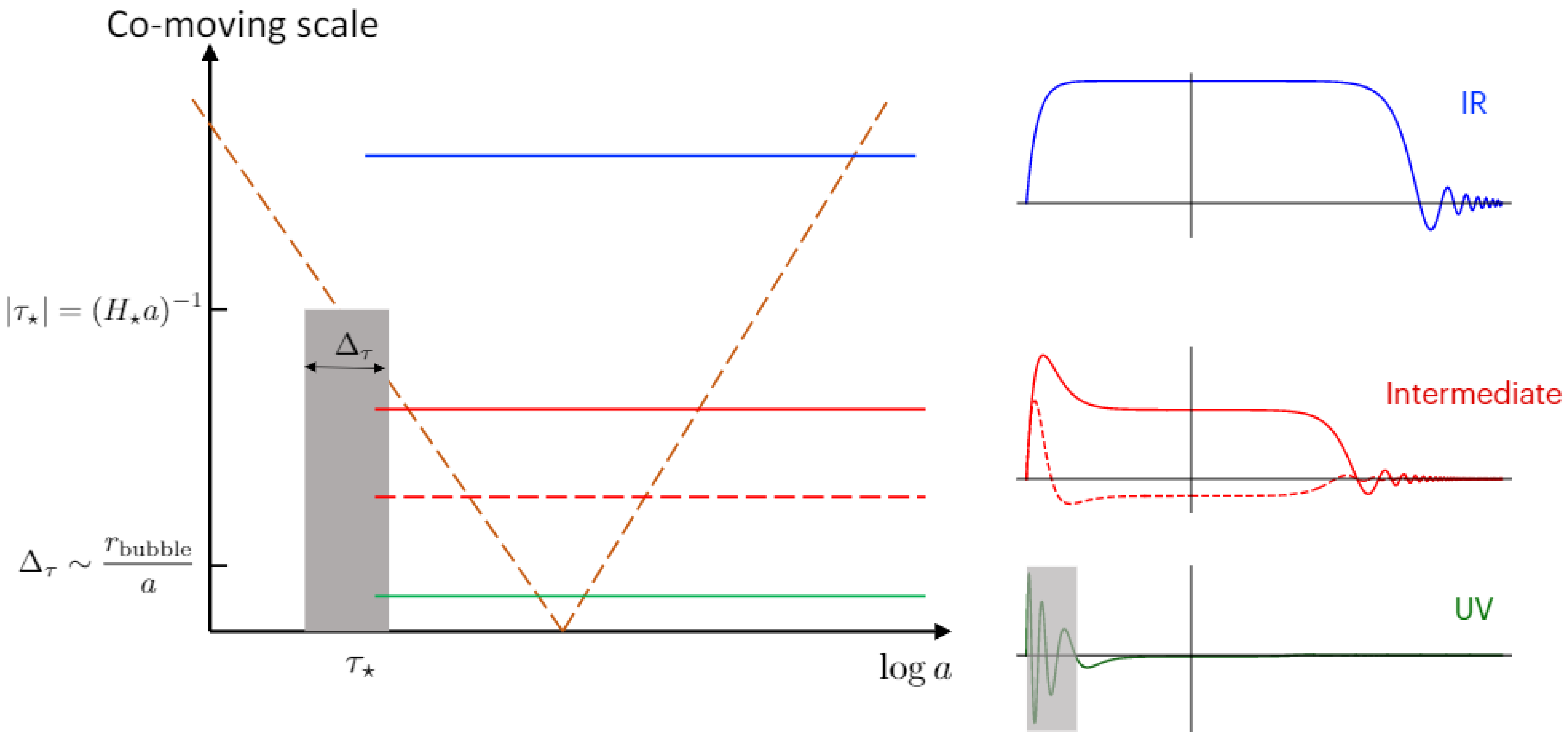
  - ☐Radiation domination

$$\mathcal{E}(\eta) = \frac{\sin\eta}{\eta} \qquad \tilde{\mathcal{E}}_0^i = \frac{f}{k}, \qquad f = a_{\rm end}^2 H_{\rm end}$$

  - ☐Matter domination

$$\mathcal{E}(\eta) = -\frac{3}{\eta^2}\left(\cos\eta - \frac{\sin\eta}{\eta}\right) \qquad \tilde{\mathcal{E}}_0^i = \frac{3f}{k^2}, \qquad f = \frac{a_{\rm end}^3 H_{\rm end}^2}{4}$$

How GW propagates in spacetime



Observational Signatures

- The observed energy density at conformal time τ

$$\rho_{\mathrm{GW}} = \frac{1}{16\pi G_N a^2}\langle h_{ij}'^2(\tau,\mathbf{x})\rangle$$

- The differential spectrum is

Sensitive to the detailed evolution of the universe when the modes re-enter the horizon

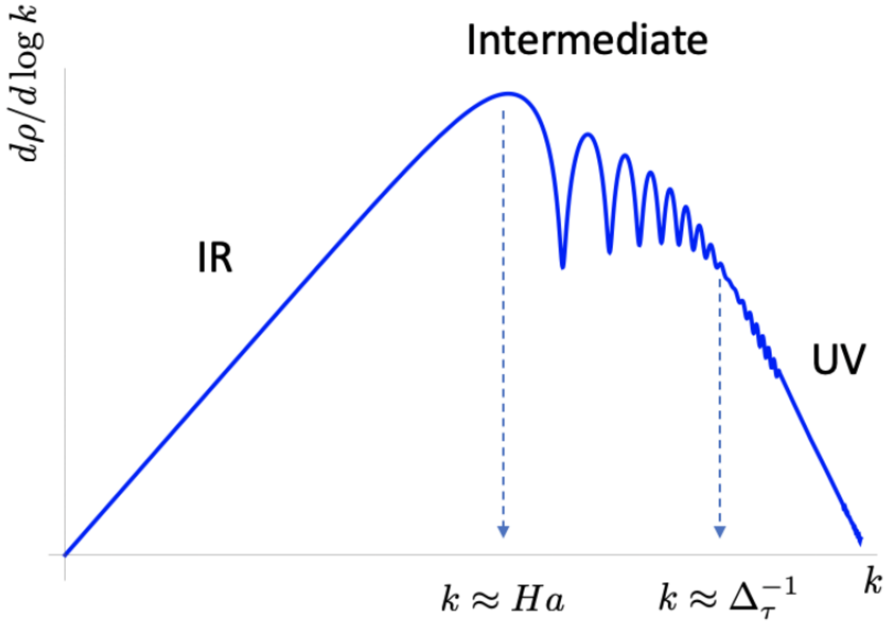
$$\frac{d\rho_{\mathrm{GW}}}{d\log k} = \frac{4G_N\left|\hat{T}_{ij}(0,0)\right|^2}{\pi V a^4(\tau)a^2(\tau_*)}\left\{\left[\tilde{\mathcal{E}}_0^i(k)\tilde{\mathcal{G}}_0^f(k)\right]^2 k^3\cos^2 k(\tau_*-\tau_0)\right\}$$

Sensitive to the detailed evolution of the universe when the modes exit the horizon

Observational Signatures

- There is oscillatory signatures in the intermediate frequency
- The slope can help us distinguish different scenarios

		RD	MD	$t^{\tilde{p}}$
UV	dS	k^{-5}	k^{-7}	$k^{-3+2\frac{p}{\tilde{p}-1}}$
	t^p	$k^{-3+2\frac{p}{1-p}}$	$k^{-5+2\frac{p}{1-p}}$	$k^{-1+2\left(\frac{p}{1-p}+\frac{p}{\tilde{p}-1}\right)}$
Intermediate		RD	MD	$t^{\tilde{p}}$
	dS	k^{-1}	k^{-3}	$k^{1+2\frac{p}{\tilde{p}-1}}$
	t^p	$k^{1+2\frac{p}{1-p}}$	$k^{-1+2\frac{p}{1-p}}$	$k^{3+2\left(\frac{p}{1-p}+\frac{p}{\tilde{p}-1}\right)}$
IR		RD	MD	$t^{\tilde{p}}$
	dS	k^3	k^1	$k^{5+2\frac{p}{\tilde{p}-1}}$
	t^p	k^3	k^1	$k^{5+2\frac{p}{\tilde{p}-1}}$



Observational Signatures

$$\Omega_{\rm GW}(k) \equiv \Omega_R \times \frac{d\rho_{\rm GW}(\tau_R)}{d\log k} \times \frac{1}{\rho_R(\tau_R)}$$

$$\Omega_{\rm GW}(k) = \Omega_R \times \frac{\Delta\rho_{\rm vac}}{\rho_{\rm inf\star}} \left[\tilde{\mathcal{E}}_0^i(k)\tilde{\mathcal{G}}_0^f(k)\right]^2 \frac{H_\star^2}{H_r^2}\left(\frac{a_\star}{a_r}\right)^4$$

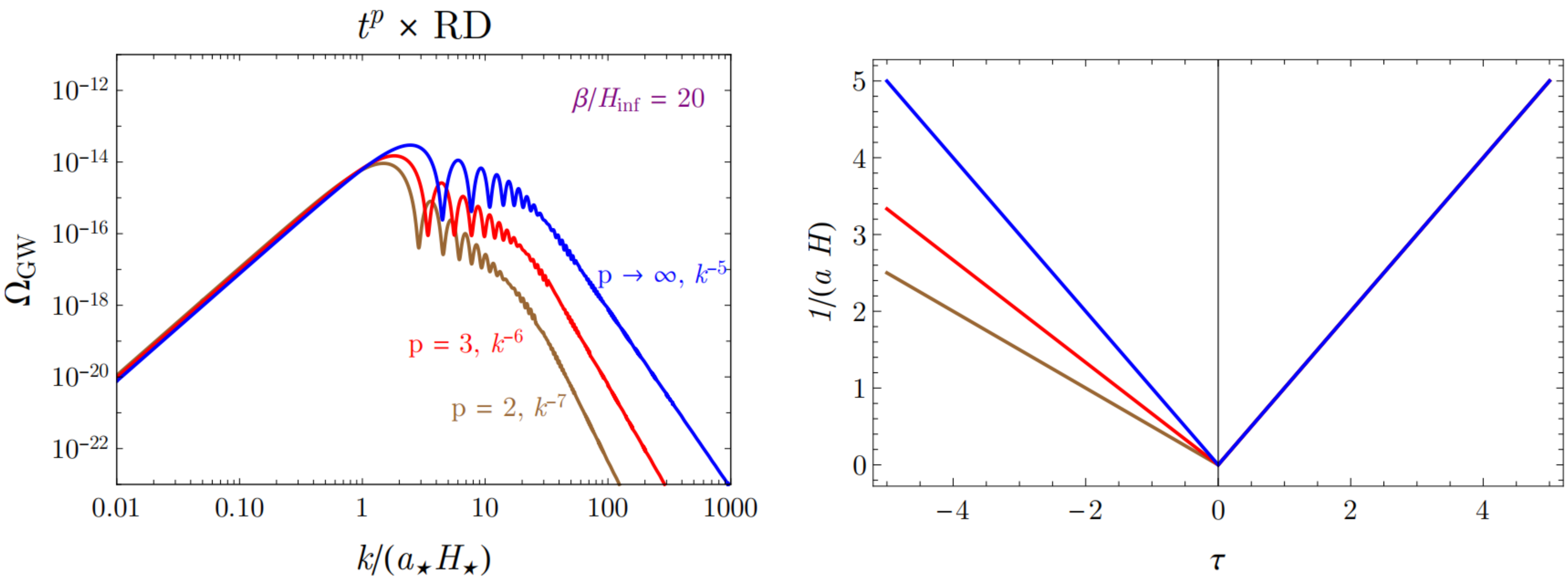
$$\times \left[1 + \mathcal{S}(k_p/\beta)\cos 2k(\tau_\star - \tau_0)\right] \times \frac{d\rho_{\rm GW}^{\rm flat}}{\Delta\rho_{\rm vac} d\log k_p}$$

Smearing

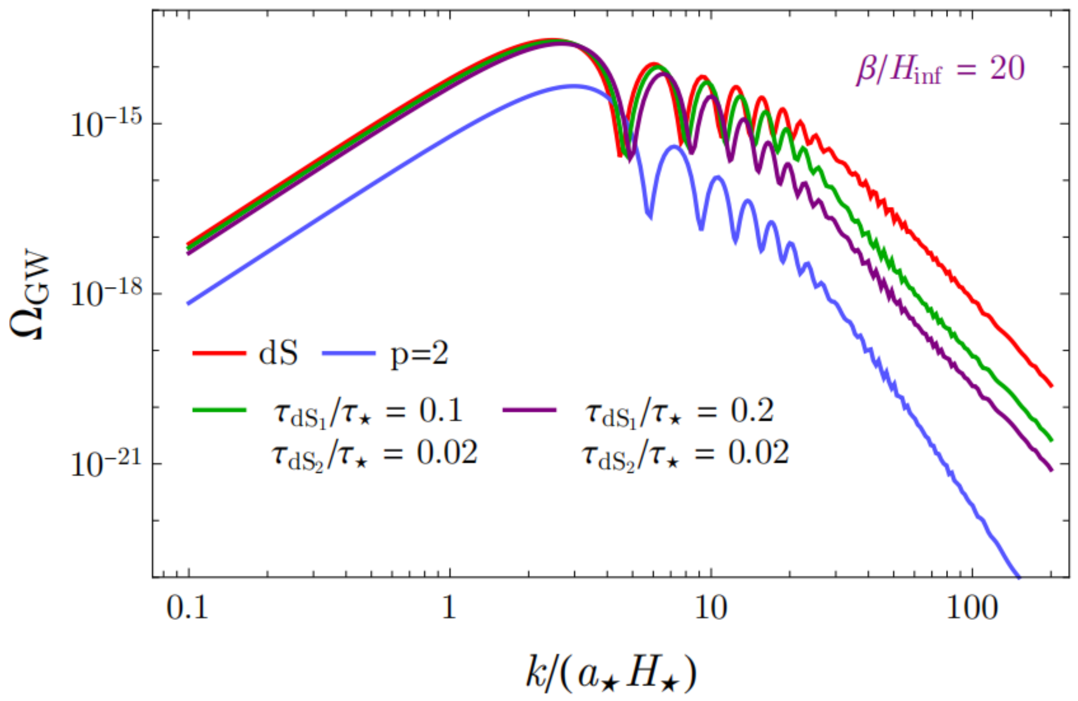
Oscillation

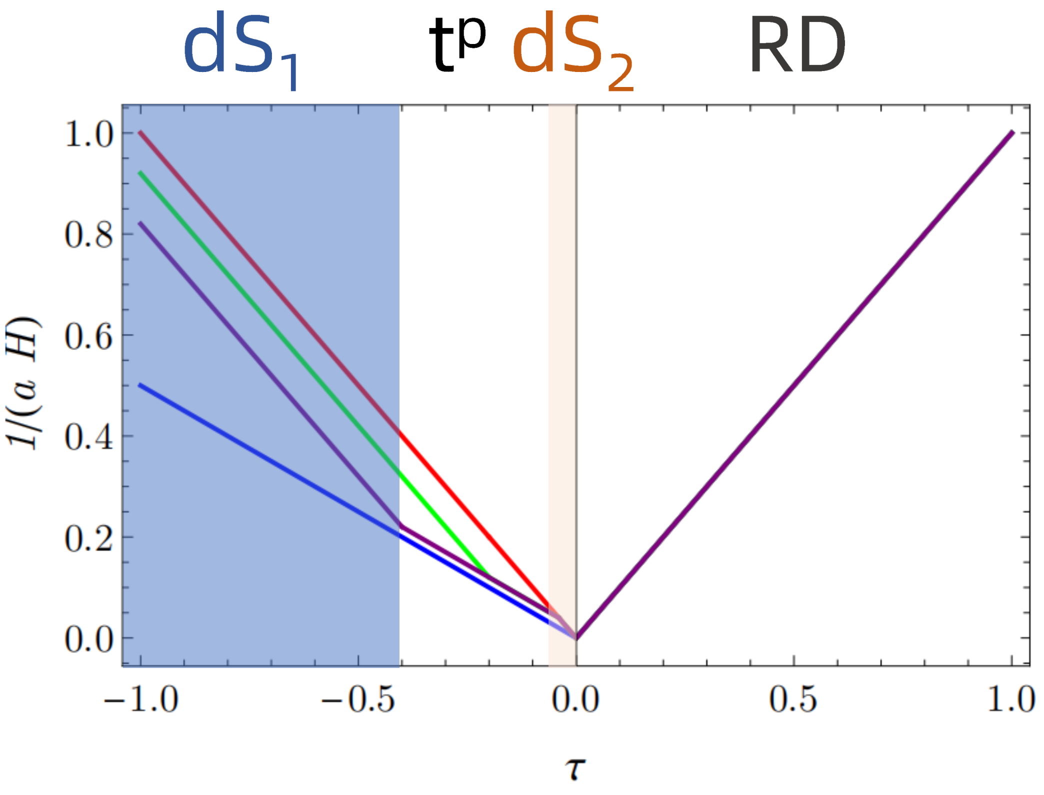
Bubble

Inflationary histories t^p vs dS



Inflationary histories two stage dS

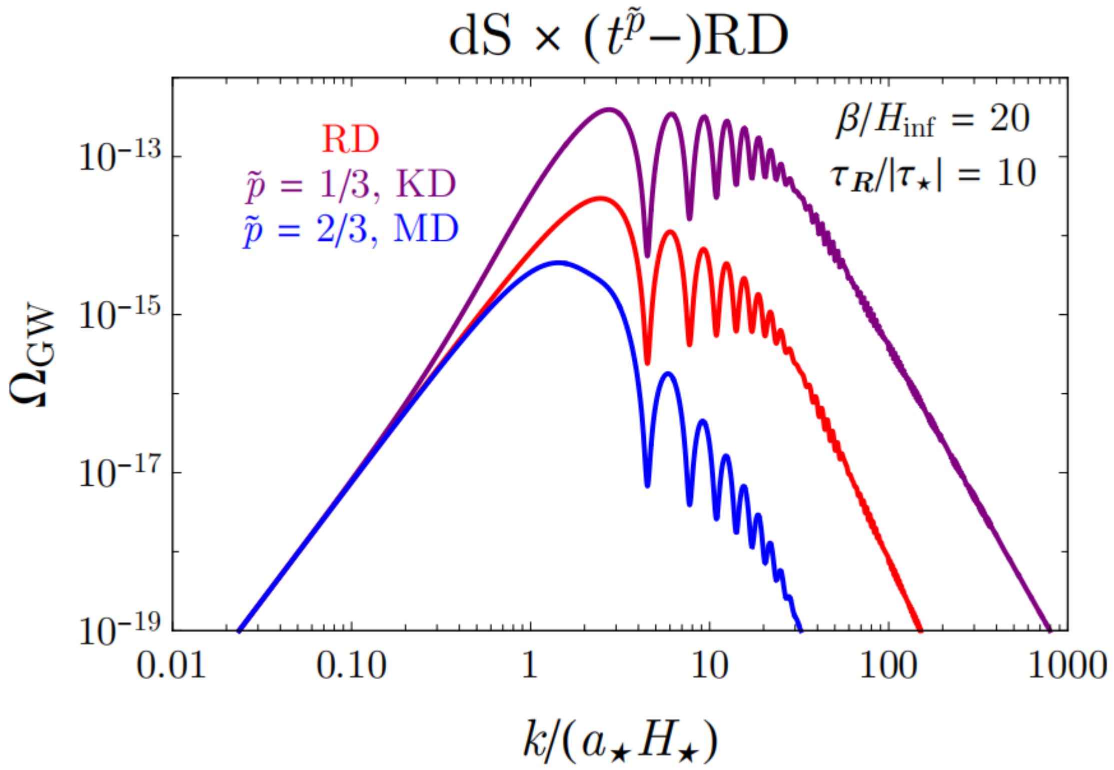


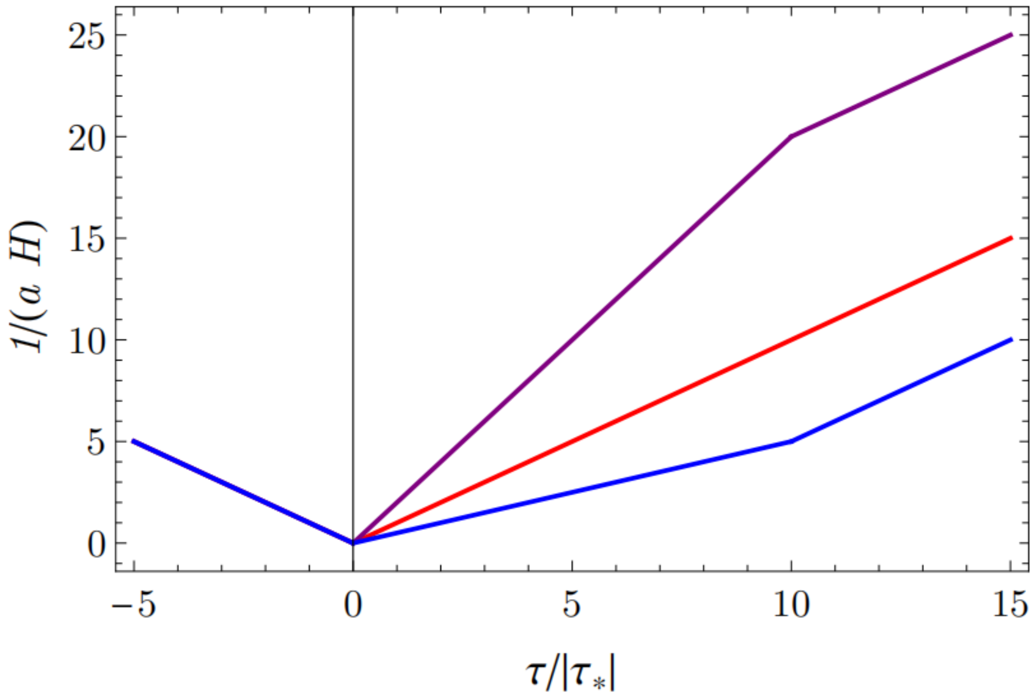


Post inflationary histories

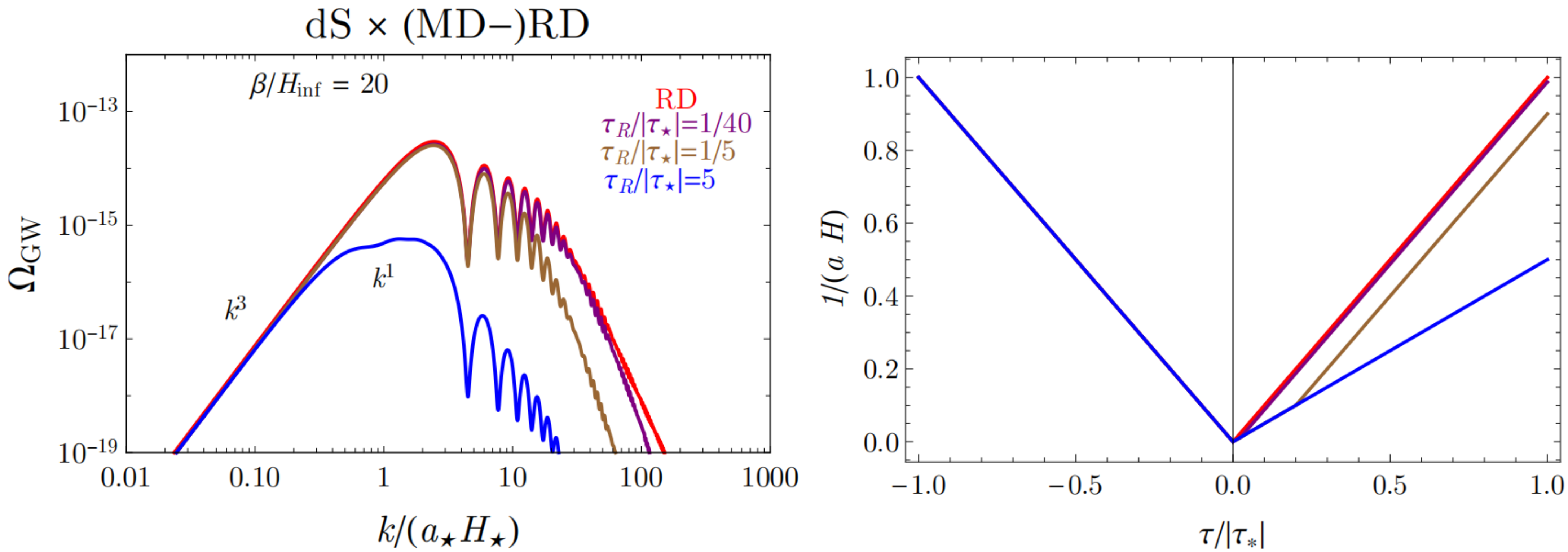
	w	$\rho(a)$	$\tilde{p}$	$\tilde{\alpha}$
kination	1	a^{-6}	1/3	0
RD	1/3	a^{-4}	1/2	-1/2
MD	0	a^{-3}	2/3	-3/2
Cosmic string	-1/3	a^{-2}	1	∞
Domain wall	-2/3	a^{-1}	2	5/2
Λ	-1	a^{0}	∞	3/2

Post inflationary histories

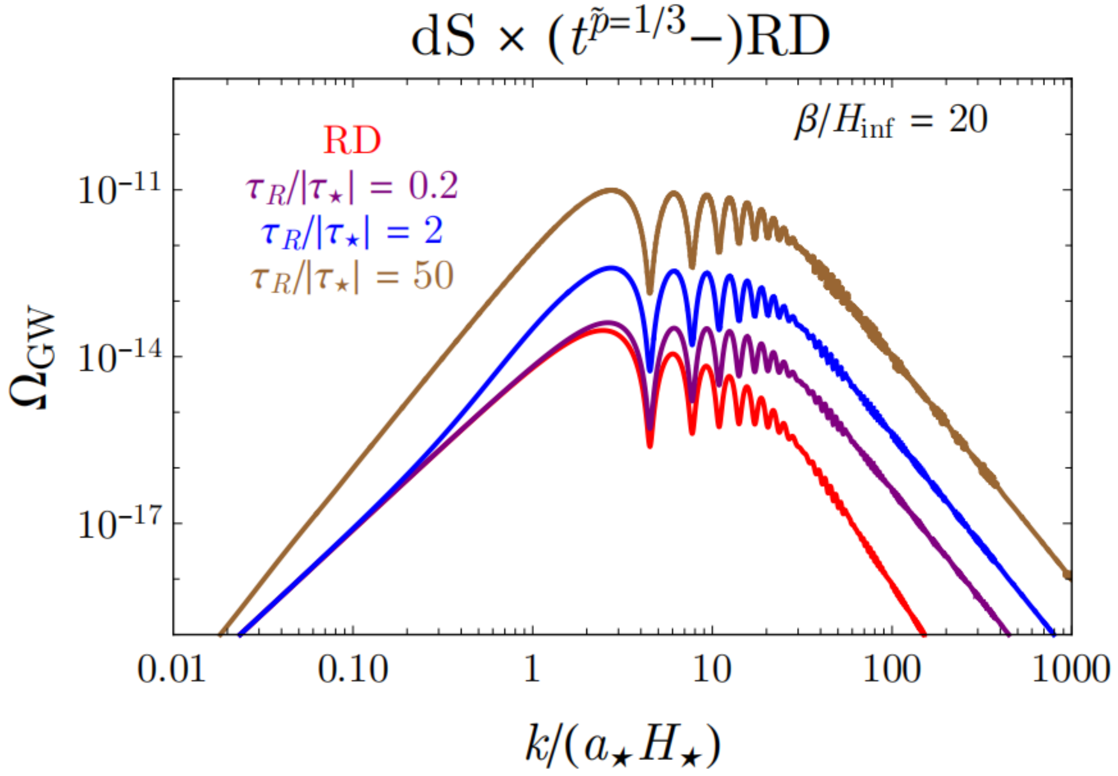


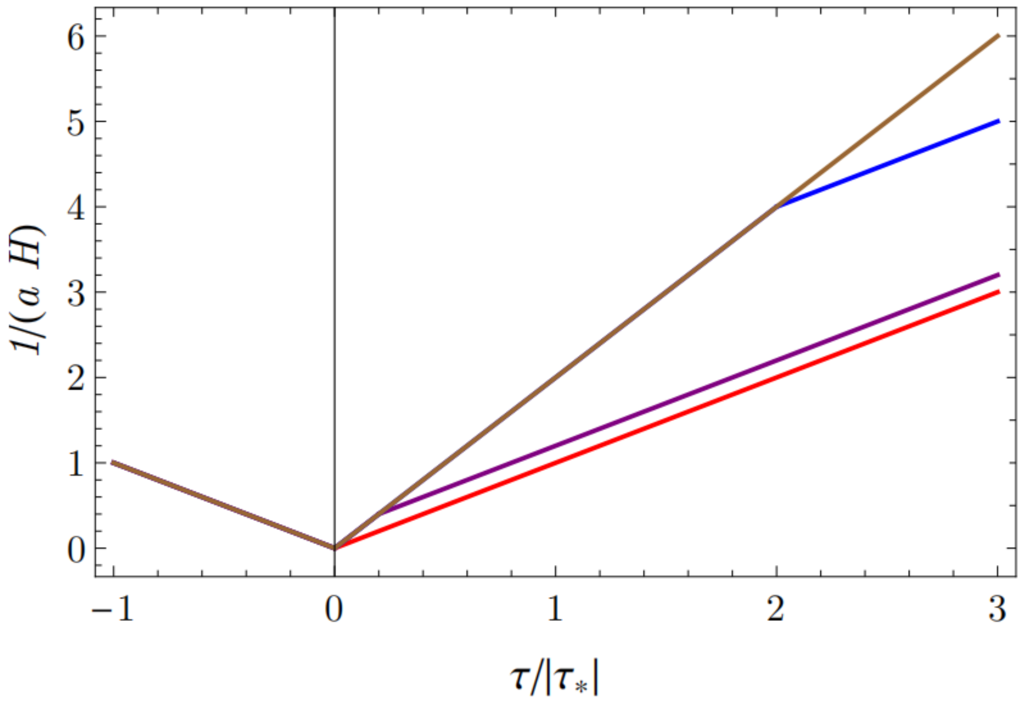


Post inflationary histories

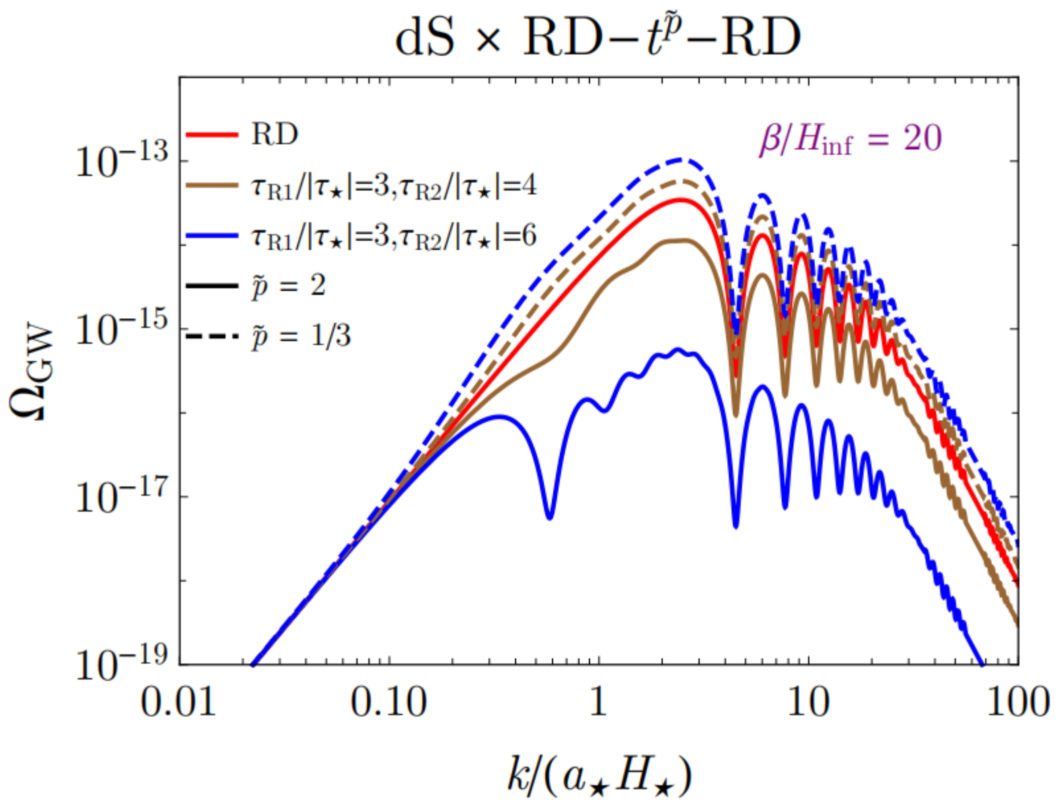


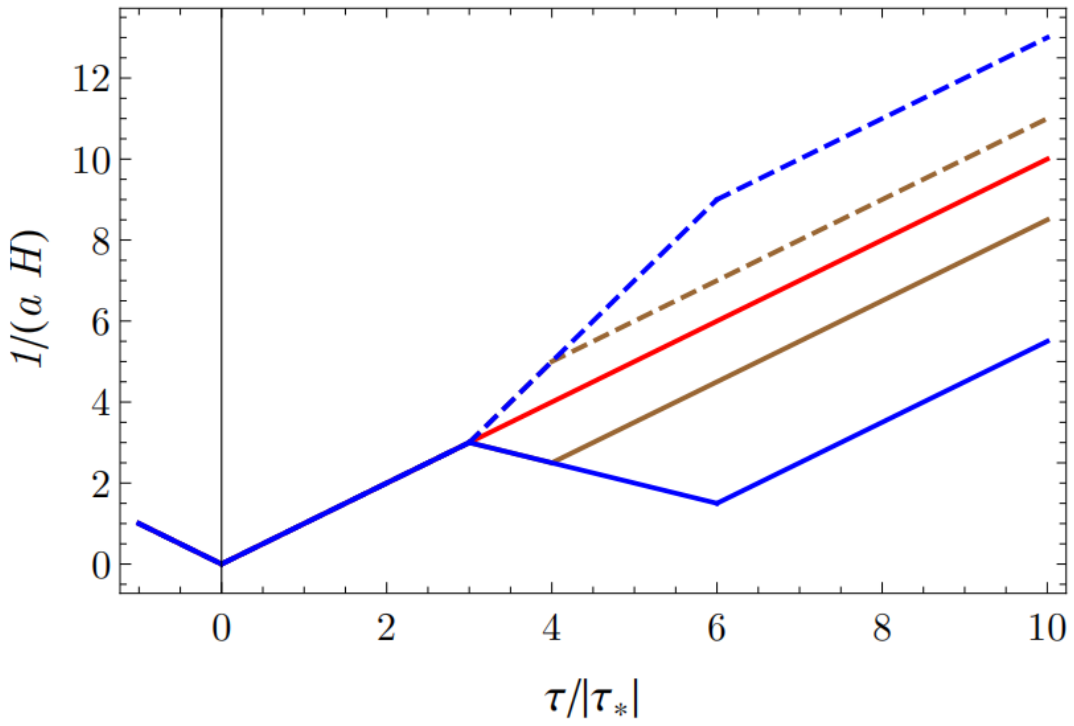
Post inflationary histories



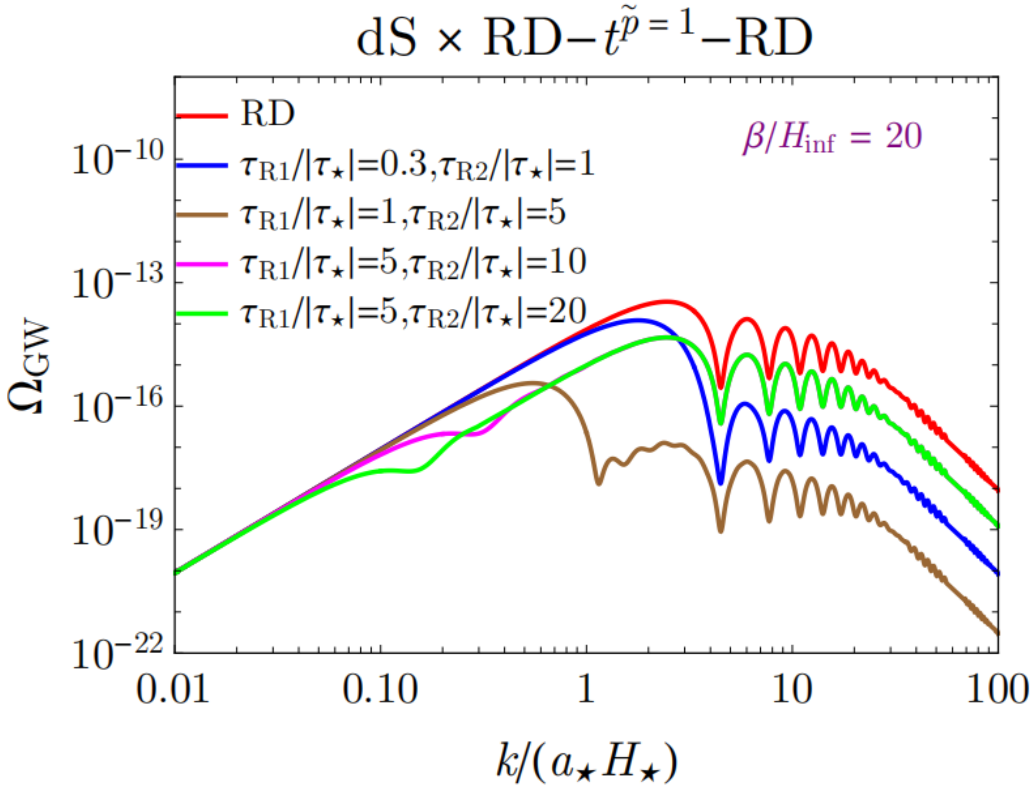


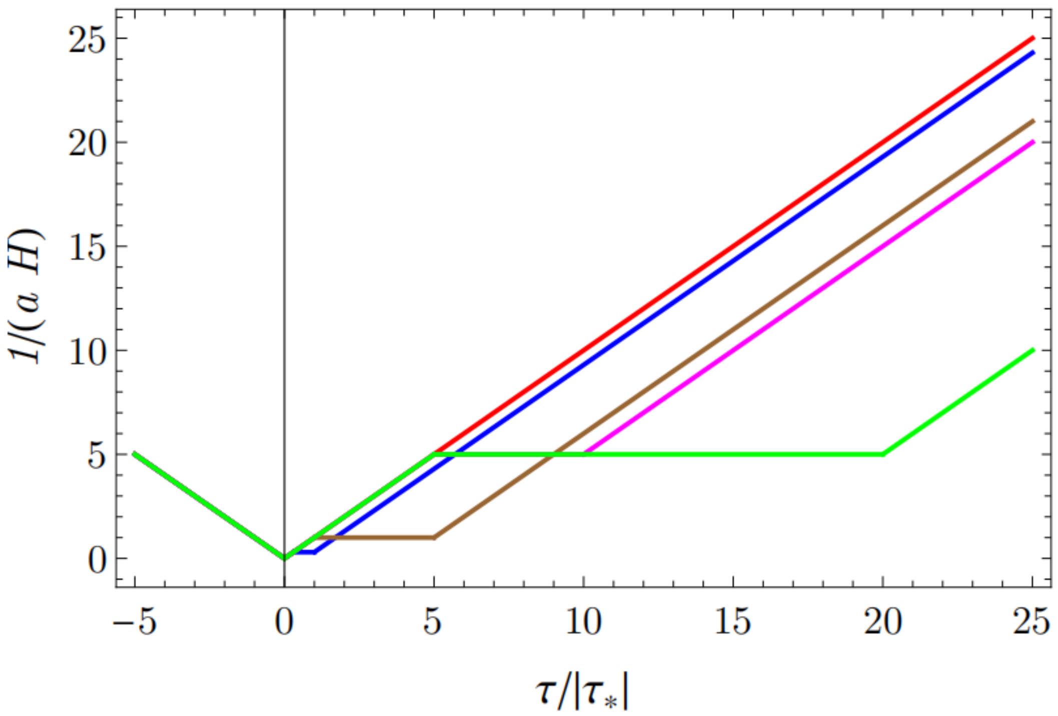
Post inflationary histories



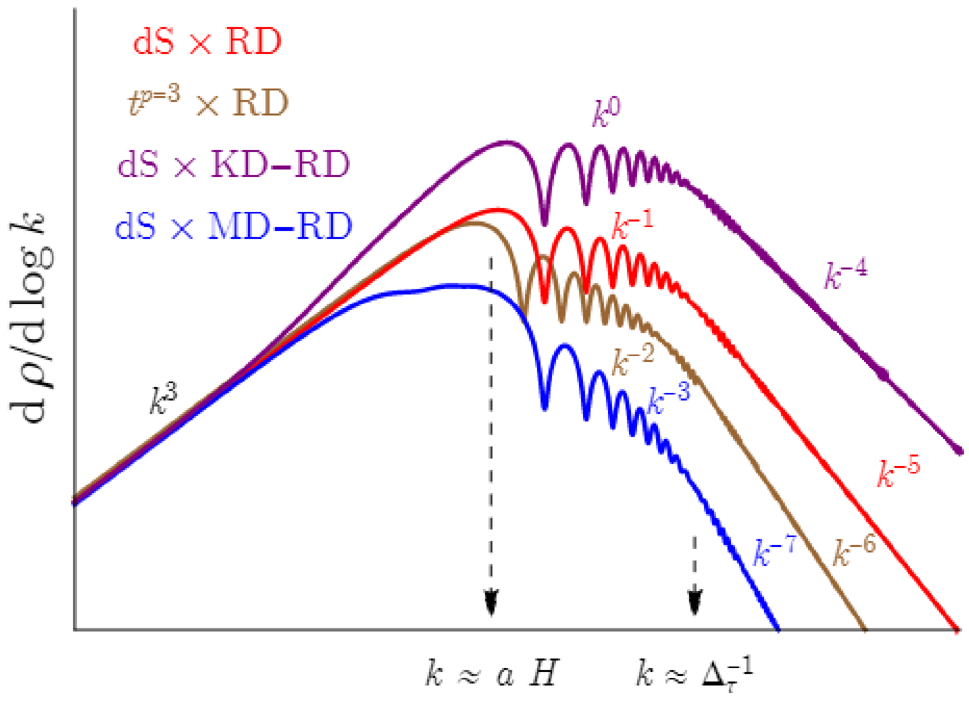


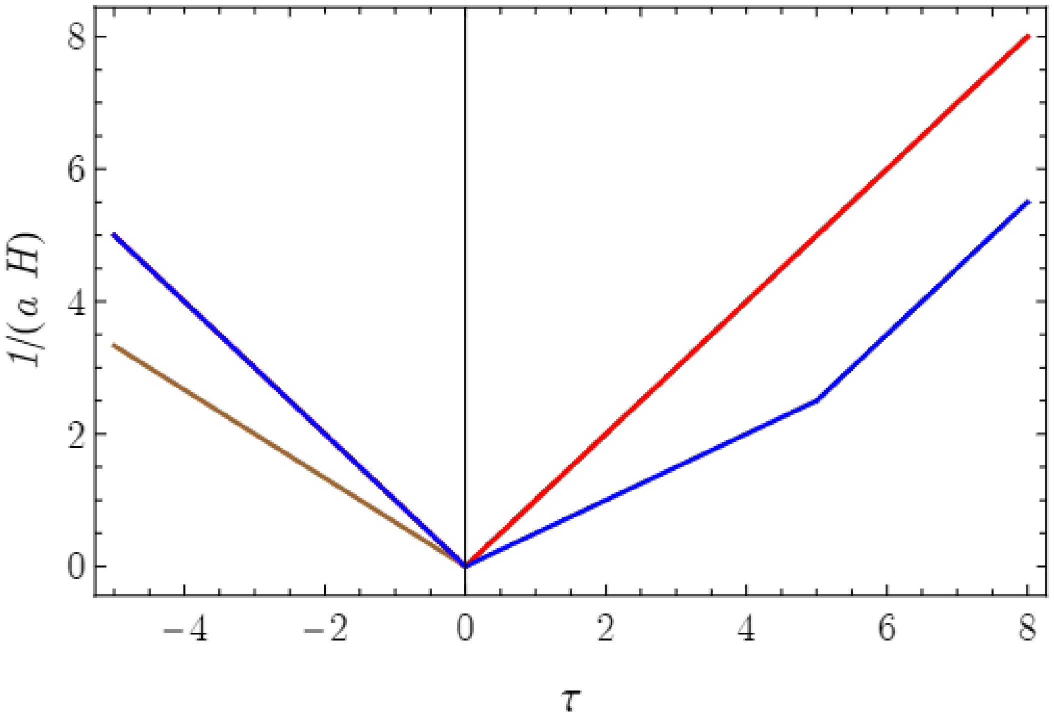
Post inflationary histories





Combined


dS × RD
$t^{p=3}$ × RD
dS × KD–RD
dS × MD–RD
$\mathrm{d}\,\rho/\mathrm{d}\log k$
k^3
k^0
k^{-1}
k^{-2}
k^{-3}
k^{-4}
k^{-5}
k^{-6}
k^{-7}
$k \approx a\ H$
$k \approx \Delta_\tau^{-1}$


1/(a H)
τ
0
2
4
6
8
−4
−2

Slope

- IR $k^3|\tilde{\mathcal{E}}_0^i|^2$ Only depends on post-inflationary histories
- Intermediate

$$\left[\tilde{\mathcal{E}}_0^i(k)\tilde{\mathcal{G}}_0^f(k)\right]^2$$

- UV

$$k^{-1}\left[\tilde{\mathcal{E}}_0^i(k)\tilde{\mathcal{G}}_0^f(k)\right]^2 \sim k^{-b-\frac{2}{p-1}-\frac{2}{1-\tilde{p}}}$$

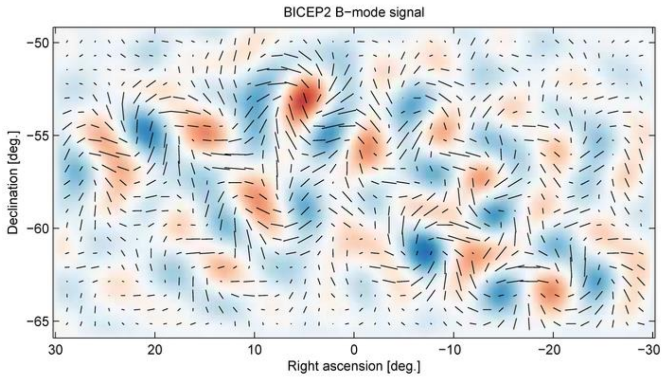
Depends on both inflationary and post-inflationary histories

Stochastic GW Background

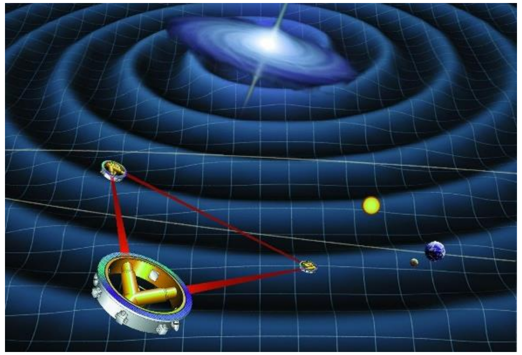
- A stochastic background of GWs can be created by the superposition of a large number of independent sources
- Analogous to the cosmic microwave background (CMB): electromagnetic (EM) record of the early universe
- A stochastic background of GWs is very different from transient GWs (binary inspirals, burst events) or continuous periodic GWs (coming from pulsars). These other sources are sending GWs from specific locations in the sky
- A stochastic background will be coming from all directions

Stochastic GW Background Experiments

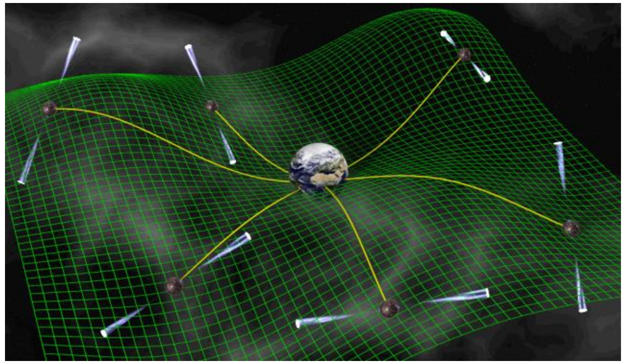
Very low frequency: CMB B mode



Middle frequency: LISA



Low frequency: Pulsar timing array



High frequency: LIGO, VIRGO

Stochastic GW Background Sources

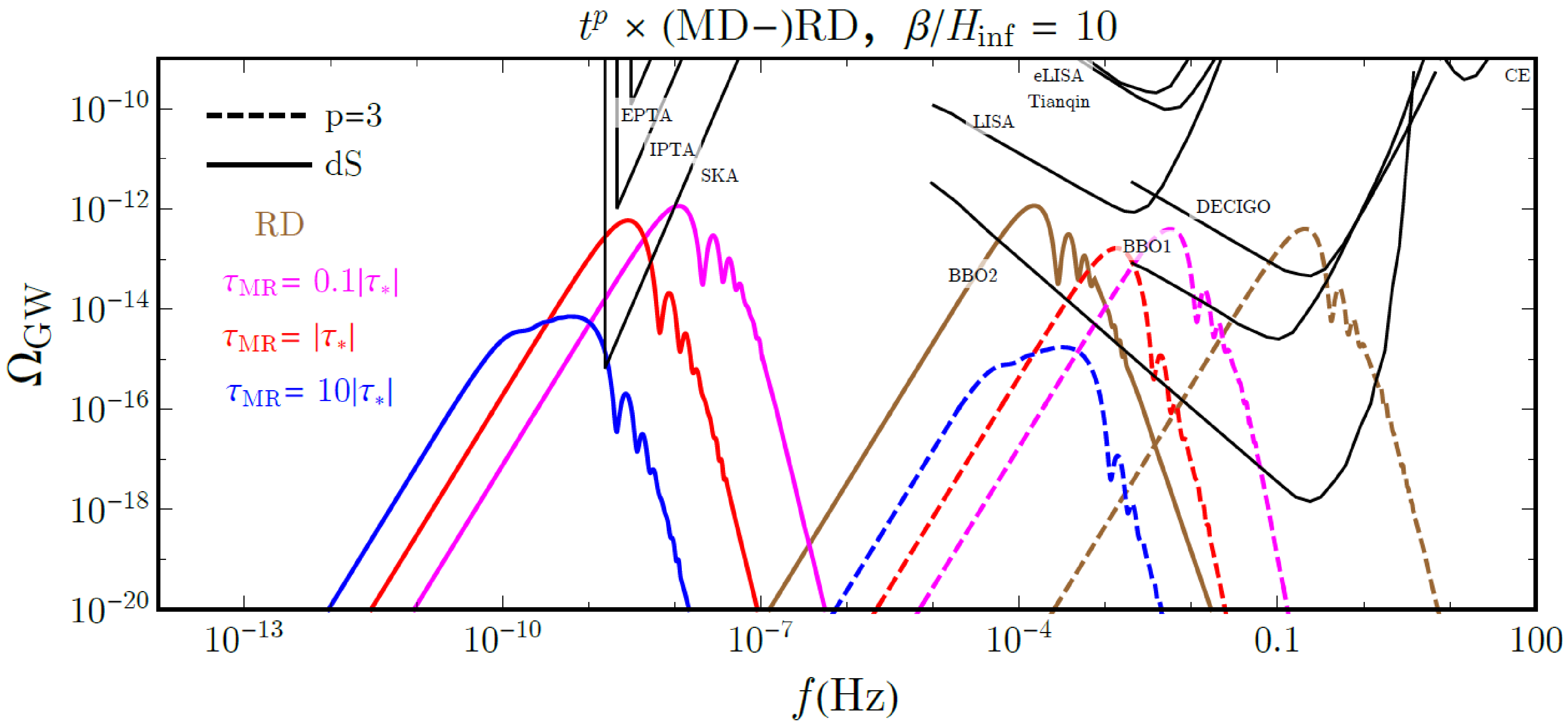
- Inflation Quantum fluctuations of GW that went outside horizon and became classical
- Cosmic strings one-dimensional topological defects
- First order phase transitions GW from bubble nucleations
- Pre big bang models an extension of the standard inflationary cosmology
- Binary black holes GWs
- Binary neutron stars GWs
- Supernovae if a supernova has some asymmetry, then GWs will be produced
- Pulsar and magnetars Non-axisymmetric spinning neutron stars are expected to be a detectable source of GWs

Observed GW frequency

$$\frac{f_{\text{today}}}{f_\star} = \frac{a(\tau_\star)}{a_1}\left(\frac{g_{*S}^{(0)}}{g_{*S}^{(R)}}\right)^{1/3} \frac{T_{\text{CMB}}}{\left[\left(\frac{30}{g_*^{(R)}\pi^2}\right)\left(\frac{3H_{\text{inf}}^2}{8\pi G_N}\right)\right]^{1/4}}$$

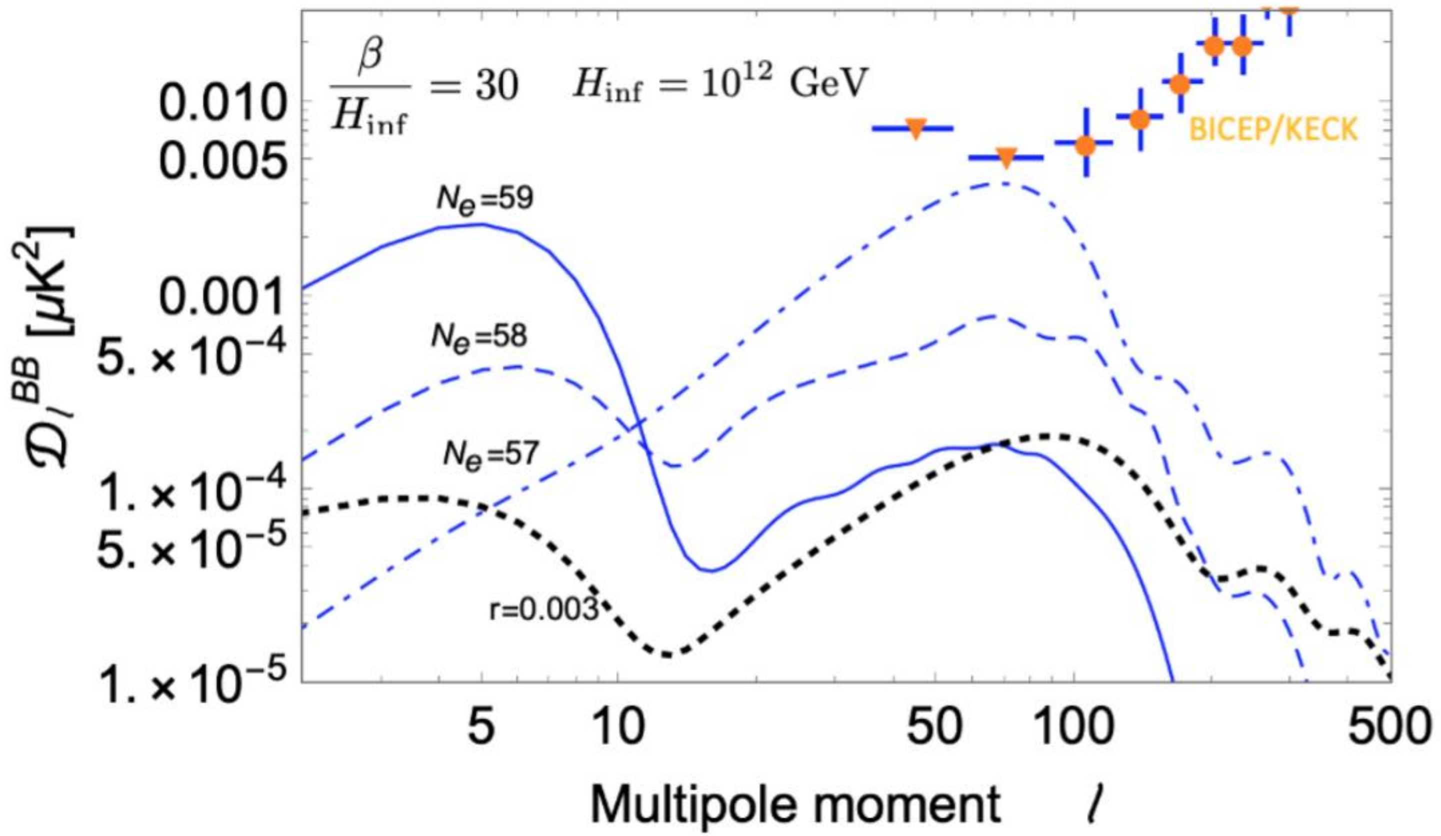
$$\tilde{f}_{\text{today}}^{\text{peak}} = 1.1\times 10^{11}\text{Hz}\times\left(\frac{H_{\text{end}}}{m_{\text{pl}}}\right)^{1/2}\left(\frac{a_r}{a_{\text{end}}}\right)^{-\frac{1}{2\tilde{\alpha}-1}-\frac{1}{2}}\frac{a\left(\tau_\star\right)}{a_{\text{end}}}$$

Observational Signatures



Observational Signatures

- B mode
- BICEP/Keck array
- CLASS package
- Small wiggles induced by the oscillatory pattern in the GW power spectrum



Conclusion

- We consider gravitational wave signatures from first order phase transition during inflation
- They can generate oscillatory signatures, which can be distinguished from gravitational waves from first order phase transition after inflation
- From the slope of the gravitational waves generated, we can infer both the inflationary history and post inflationary history