Spherically symmetric scalar hair for charged BHs
- a counter example of scalar no-hair theorem -

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in collaboration with

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Based on

PLB 803 (2020) 135324 (gr-qc/1907.04982)

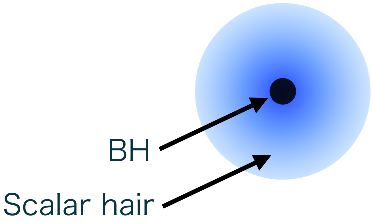
PRL 125 (2020) 11, 111104 (gr-qc/2004.03148)

TOHOKU UNIVERSITY

October 26th, 2021 - Gravity & Cosmology Seminar at ITP-CAS

Outline

- No-scalar-hair theorem
- Examples of scalar hair for charged BHs
- Probe limit and Q-ball
- Summary



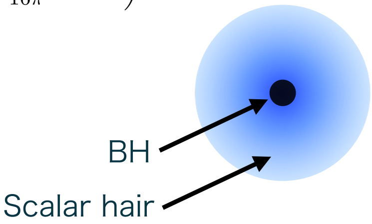
No-hair theorem for black holes

- A black hole (BH) can be characterized only by mass, angular momentum, and charge.

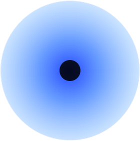
Israel '67, Carter '71, Ruffini & Wheeler '71

- However, one may ask if scalar hair can exist around a BH.

$$S = \int \sqrt{-g} d^4x \left(-\frac{1}{16\pi} R - (D^\alpha \Phi)^* D_\alpha \Phi - V(\Phi) - \frac{1}{16\pi} F^{\alpha\beta} F_{\alpha\beta} \right)$$

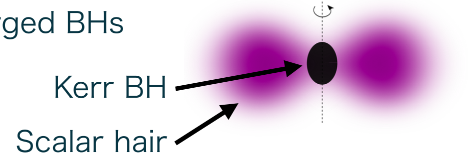


No-scalar-hair theorem for black holes



- Some examples of scalar hair:
  - Skyrmion hair — Luckock & Moss '86, Droz, Heusler, Straumann '91
  - Axion hair — Campbell, Kaloper, Olive '91, Mignemi, Stewart '93, Campbell, Duncan, Kaloper, Olive '90, Duncan, Kaloper, Olive '92
  - Dilaton hair — Kanti, Mavromatos, Rizos, Tamvakis, Winstanley '96
  - Scalar hair around Kerr BH — Hod '12, '14, Herdeiro, Radu '14, Benone, Crispino, Herdeiro, Radu '14, Huang, Liu '16, Huang, Liu, Zhai, Li '17

- No examples of (simple) scalar hair for charged BHs



No-scalar-hair theorem for black holes

Bekenstein '95

- Bekenstein '95:

  The theorem rules out a multicomponent scalar field dressing of any asymptotically flat, static, spherically symmetric black hole. The field is assumed to be minimally coupled to gravity and to bear a non-negative energy density as seen by any observer.

- In short, spherically symmetric BHs cannot have (singlet) scalar hair.
- This theorem can apply only to a system with $T^{tt} = T^{\theta\theta}$. For example, a charged scalar field do not satisfy this relation.

No-scalar-hair theorem for black holes

Mayo & Bekenstein '96

- Mayo & Bekenstein '96:

  There exists no non-extremal static and spherical charged black hole endowed with hair in the form of a charged scalar field, whether minimally or non-minimally coupled to gravity, and with a regular positive semidefinite self-interaction potential.

- In short, charged BHs cannot have (charged) scalar hair.
- This does not apply to Kerr BHs, which are not spherically symmetric.

No-scalar-hair theorem for black holes

- Some examples of scalar hair:
  - Skyrmion hair — Luckock & Moss '86, Droz, Heusler, Straumann '91
  - Axion hair — Campbell, Kaloper, Olive '91, Mignemi, Stewart '93, Campbell, Duncan, Kaloper, Olive '90, Duncan, Kaloper, Olive '92
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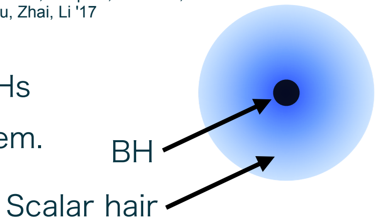
- No examples of (simple) scalar hair for charged BHs
- These are consistent with the no-scalar-hair theorem.



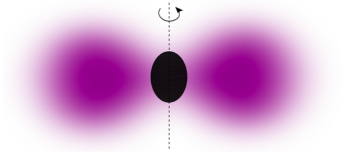
No-scalar-hair theorem for black holes

- Kerr (rotating) BHs can have scalar hair.
  - What prevents the scalar field from being absorbed into the BH?

    ← Angular momentum $V_{\text{eff}} \propto 1/r^2$

  - What makes the scalar field locarized?

    ← Gravitational interaction $V_{\text{grav}} \propto -1/r$



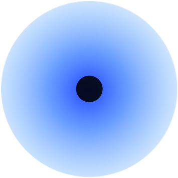
No-scalar-hair theorem for black holes

- Let us think about a charged scalar hair around a charged BH.
  - What prevents the scalar field from being absorbed into the BH?

    ← Gauge interaction $V_{\text{electric}} \propto 1/r$

  - What makes the scalar field locarized?

    ← ~~Gravitational interaction~~ $V_{\text{grav}} \propto -1/r$



No-scalar-hair theorem for black holes

Hong, Suzuki, **MY** '19, '20

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    ← Gauge interaction $V_{\mathrm{electric}} \propto 1/r$

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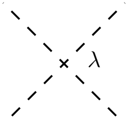
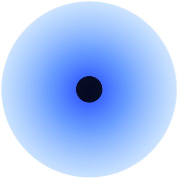
    Self interaction of scalar field



$\lambda < 0$: Attractive

$\lambda > 0$: Repulsive



No-scalar-hair theorem for black holes

Hong, Suzuki, **MY** '19, '20

- It seems that the two necessary conditions can be satisfied for a self-interacting charged scalar field around a charged BH.

- On the other hand, the no-scalar-hair theorem rules out hairly solutions.
- Actually, we have found that the theorem is not correct.

- Their EoM at a large distance:

$$\varphi_{,rr} + \frac{2}{r}\varphi_{,r} + q^2 A_t(\infty)^2 \varphi \overset{?}{\approx} 0$$

- The total energy diverges:

$$\varphi \approx \frac{1}{r} \sin\left(qA_t(\infty) r + \chi\right)$$

Mayo & Bekenstein '96

- Correct EoM at a large distance

$$\varphi_{,rr} + \frac{2}{r}\varphi_{,r} - \left(V''(0) - q^2 A_t(\infty)^2\right)\varphi \approx 0$$

- The total energy is finite:

$$\varphi \sim \frac{1}{r} e^{-\sqrt{V''(0) - q^2 A_t^2(\infty)}\, r}$$

Hong, Suzuki, **MY** '19, '20

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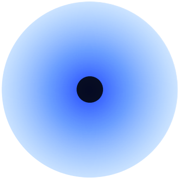
Scalar hair for charged BHs

Hong, Suzuki, **MY** '19, '20

- We have found hairy solutions around charged BHs.

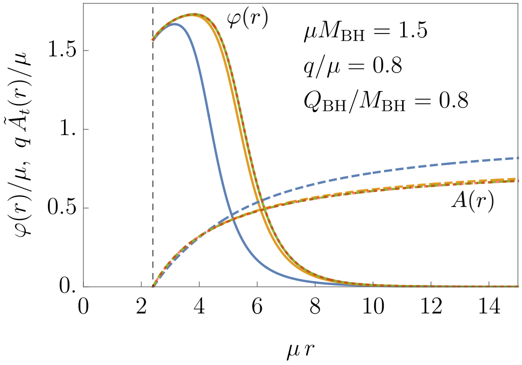
$$S = \int \sqrt{-g} d^4x \left(-\frac{1}{16\pi} R - (D^\alpha \Phi)^* D_\alpha \Phi - V(\Phi) - \frac{1}{16\pi} F^{\alpha\beta} F_{\alpha\beta} \right)$$



$$V(\Phi) = \mu^2 |\Phi|^2 - \kappa |\Phi|^4 + A|\Phi|^6$$

$$A = \kappa^2 / 3\mu^2$$

$$\mu = 0.02,\ 0.005,\ 0.001$$



Scalar hair for charged BHs

Hong, Suzuki, **MY** '19, '20

- We have found hairy solutions around charged BHs.
- If the gauge interaction is strong enough, the scalar configuration becomes a shell-like solution.

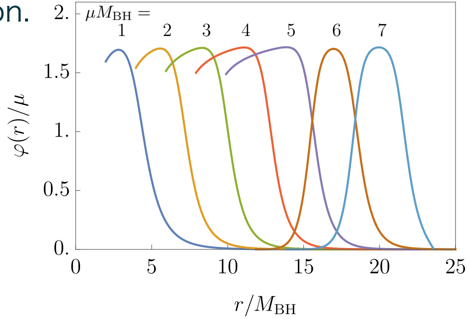
$$S = \int \sqrt{-g} d^4x \left(-\frac{1}{16\pi} R - (D^\alpha \Phi)^* D_\alpha \Phi - V(\Phi) - \frac{1}{16\pi} F^{\alpha\beta} F_{\alpha\beta} \right)$$

$$V(\Phi) = \mu^2 |\Phi|^2 - \kappa |\Phi|^4 + A |\Phi|^6$$

$$A = \kappa^2 / 3\mu^2$$

$$q/\mu = 9$$

$$Q_{\rm BH}/M_{\rm BH} = 0.2$$



Scalar hair for charged BHs

Hong, Suzuki, **MY** '19, '20

- We have found hairy solutions around charged BHs.
- The length of hair can be very small in the extremal limit of BHs.

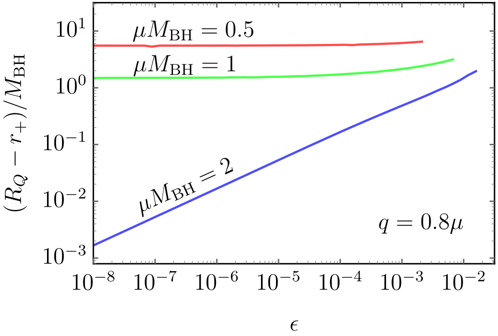
$$S = \int \sqrt{-g} d^4x \left(-\frac{1}{16\pi} R - (D^\alpha \Phi)^* D_\alpha \Phi - V(\Phi) - \frac{1}{16\pi} F^{\alpha\beta} F_{\alpha\beta} \right)$$

$$V(\Phi) = \mu^2 |\Phi|^2 - \kappa |\Phi|^4 + A |\Phi|^6$$

$$A = \kappa^2 / 3\mu^2$$

$$q/\mu = 0.8$$

$$Q_{\rm BH} = (1-\epsilon) M_{\rm BH}$$



Scalar hair for charged BHs

Hong, Suzuki, **MY** '19, '20

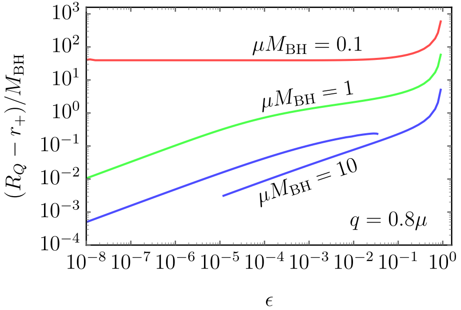
- We have found hairy solutions around charged BHs.
- We also investigated a scalar field with a logarithmic potential, which is motivated by supersymmetric models.

$$S = \int \sqrt{-g} d^4x \left(-\frac{1}{16\pi} R - (D^\alpha \Phi)^* D_\alpha \Phi - V(\Phi) - \frac{1}{16\pi} F^{\alpha\beta} F_{\alpha\beta} \right)$$

$$V(\Phi) = \mu^4 \ln \left(1 + |\Phi|^2/\mu^2\right)$$

$$q/\mu = 0.8$$

$$Q_{\rm BH} = (1-\epsilon) M_{\rm BH}$$



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Probe limit and Q-ball

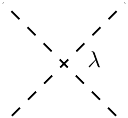
- Let us look at the two necessary conditions in more detail.
  - What prevents the scalar field from being absorbed into the BH?

    ← Gauge interaction $\propto \frac{1}{r}$

  - What makes the scalar field locarized?

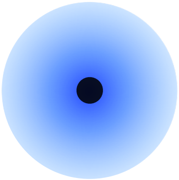
    ← ~~Gravitational interaction~~ $\propto \frac{1}{r}$

    Self interaction of scalar field



$\lambda < 0$: Attractive

$\lambda > 0$: Repulsive



Probe limit and Q-ball

Hong, Suzuki, **MY** '19, '20

- What prevents the scalar field from being absorbed into the BH?
  - Equation of motion near the BH surface is given by

$$\frac{d^2\varphi}{dr_*^2} + (\omega_\Phi - \omega_c)^2 \varphi \approx 0 \qquad \Phi = \varphi(r) e^{-i\omega_\Phi} \qquad \omega_c \equiv \frac{qQ_{\rm BH}}{r_H}$$

  - To prevent the absorption, we require $\omega_\Phi = \omega_c$

Probe limit and Q-ball

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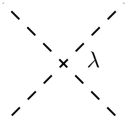
  - To prevent the absorption, we require $\omega_\Phi = \omega_c$
  - The phase velocity ω_Φ can be identified as a chemical potential of the scalar hair and hence is equal to the energy of scalar field per unit charge. $\omega_\Phi = q\frac{dE_\Phi}{dQ_\Phi}$
  - The parameter ω_c is the electrostatic energy of BH per unit charge.

Probe limit and Q-ball

Hong, Suzuki, **MY** '19, '20

- What makes the scalar field locarized?

  ← Self interaction of scalar field



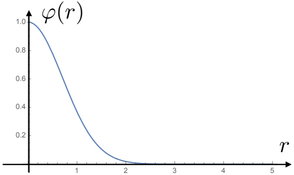
$\lambda < 0$: Attractive

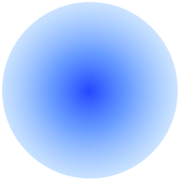
$\lambda > 0$: Repulsive

- Actually, there exist localized configurations of scalar field, called Q-balls, even in the vacuum without a BH if the scalar field has an attractive self-interaction.

Coleman '85

$$\mathrm{Min}_{\varphi}\left[\frac{2V(\varphi)}{\varphi^2}\right] < \omega_{\Phi}^2 < \frac{\partial^2 V(0)}{\partial \varphi^2}$$

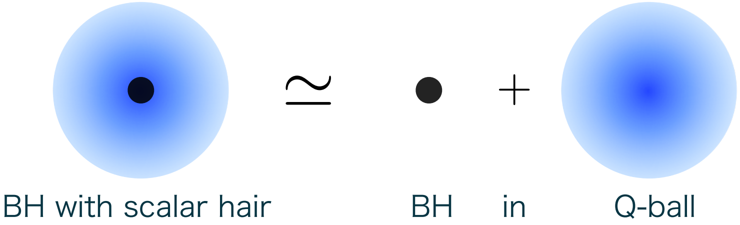




Probe limit and Q-ball

Hong, Suzuki, **MY** '19, '20

- In the limit of $Q_{\rm BH} \gg Q_\Phi,\ M_{\rm BH} \gg E_\Phi$, we can neglect the backreaction of scalar hair to the BH. This is called a probe limit.
- In the limit of long hair, the gravitational effect on the scalar hair is negligible.
- In these limits, we can construct a hairy solution by putting a BH inside a Q-ball. We still need to fulfill the condition of $\omega_\Phi = \omega_c$ to prevent absorption.



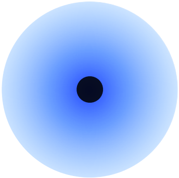
Probe limit and Q-ball

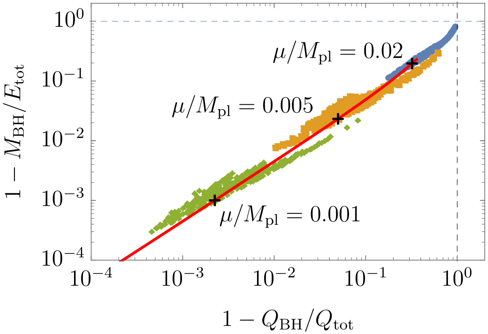
Hong, Suzuki, **MY** '19, '20

- We have found hairy solutions around charged BHs.
- In the limit of small $\mu/M_{\rm Pl}$, we find $Q_{\rm BH} \gg Q_\Phi$, $M_{\rm BH} \gg E_\Phi$ and the backreaction to BH is negligible.

$$V(\Phi) = \mu^2|\Phi|^2 - \kappa|\Phi|^4 + A|\Phi|^6$$

$$A = \kappa^2/3\mu^2$$

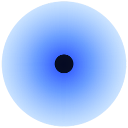


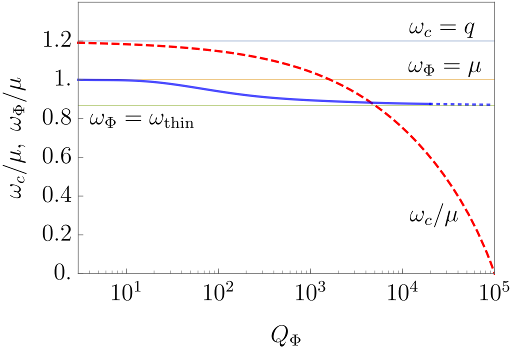


Probe limit and Q-ball

Hong, Suzuki, **MY** '19, '20

- We can discuss the stability of scalar hair in these limits.
  - blue line: $\omega_\Phi = q \dfrac{dE_\Phi}{dQ_\Phi}$
  - red dashed line: $\omega_c \equiv \dfrac{qQ_{\rm BH}}{r_H}$
- $\omega_\Phi = \omega_c$ is satisfied at the intersection,
- It is actually an attractor solution.





$Q_{\rm tot} = Q_{\rm BH} + Q_\Phi = ({\rm const.})$

Summary

- Mayo & Bekenstein '96:

  There exists no non-extremal static and spherical charged black hole endowed with hair in the form of a charged scalar field, whether minimally or non-minimally coupled to gravity, and with a regular positive semidefinite self-interaction potential.

- A charged BH can have scalar hair. It is very simple because of O(3) symmetry.
- Under a certain limit, it can be regarded as a BH surrounded by a Q-ball.

Comments and future directions

- Our scalar hair is secondary in the sense that it does not introduce new physical parameters in the solutions. The hair is not an independent parameter from the mass and charge.
- The scalar hair may affect the BH shadow, which can be observed by Event Horizon Telescope.
- One may apply AdS/CFT to the near-extremal BH with a scalar hair.
- There may be some implication from the string theory because Reissner-Nordstrom BH can be identified as a D-brane in certain spacetime dimensions.
- Does the no-Cauchy-horizon theorem apply? e.g., Cai, Li, Yang '20, An, Li, Yang '21

The field equations and the Einstein equations are written as

$$\varphi_{,rr} + \frac{1}{2}\left(\frac{4}{r} + \nu_{,r} - \lambda_{,r}\right)\varphi_{,r} - (V' - q^2 e^{-\nu} A_t^2) e^{\lambda}\varphi = 0 \ , \tag{2.3}$$

$$A_{t,rr} + \frac{1}{2}\left(\frac{4}{r} - \nu_{,r} - \lambda_{,r}\right) A_{t,r} - 4\pi q^2 \varphi^2 e^{\lambda} A_t = 0 \ , \tag{2.4}$$

$$e^{-\lambda}\left(\frac{1}{r^2} - \frac{\lambda_{,r}}{r}\right) - \frac{1}{r^2} = 8\pi T_t^t \ , \tag{2.5}$$

$$e^{-\lambda}\left(\frac{\nu_{,r}}{r} + \frac{1}{r^2}\right) - \frac{1}{r^2} = 8\pi T_r^r \ , \tag{2.6}$$

where $V' \equiv \partial V/\partial\varphi$ and (t,t) and (r,r) components of the energy-momentum tensor are given by

$$8\pi T_t^t = 4\pi\left(-e^{-\lambda}\varphi_{,r}^2 - e^{-\nu} q^2 A_t^2\, \varphi^2 - V\right) - e^{-\nu-\lambda} A_{t,r}^2, \tag{2.7}$$

$$8\pi T_r^r = 4\pi\left(e^{-\lambda}\varphi_{,r}^2 + e^{-\nu} q^2 A_t^2\, \varphi^2 - V\right) - e^{-\nu-\lambda} A_{t,r}^2. \tag{2.8}$$

- Boundary conditions:

$$\nu(\infty) = 0, \quad \lambda(\infty) = 0 \;\; \text{or} \;\; E(r_H) = \frac{r_H}{2},$$
$$A_t(r_H) = 0, \quad \varphi(\infty) = 0,$$
$$\varphi_{,r}(r_\mathrm{H}) = \frac{\varphi_0 V'(\varphi_0)}{r_\mathrm{H}\left(\frac{1}{r_\mathrm{H}^2} + 4\pi\left(-V(\varphi_0) - \frac{Q_\mathrm{BH}^2}{4\pi r_\mathrm{H}^4}\right)\right)} ,$$

- Behavior near the BH horizon:

$$e^\nu = c_\nu(r - r_\mathrm{H}) + \mathcal{O}((r - r_H)^2) , \quad e^\lambda = \frac{c_\lambda}{r - r_\mathrm{H}} + \mathcal{O}((r - r_H)^0) ,$$
$$A_t = c_A - c_A'(r - r_\mathrm{H}) + \mathcal{O}((r - r_H)^2) ,$$

$$e^{-\lambda} \approx 1 - \frac{2M_\mathrm{BH}}{r} + \frac{Q_\mathrm{BH}^2}{r^2} - \frac{8\pi\Lambda r^2}{3}, \qquad \text{for } r \approx r_\mathrm{H},$$